TOA-based Passive Position Estimation by Considering Time Offset

Jian ZHANG1,2,∗, Ping CAO3

1School of Air Transportation, Shanghai University of Engineering Science, Shanghai 201620, China
2School of Computer Science, Fudan University, Shanghai 200433, China
3School of Economics and Management, Fuzhou University, Fuzhou 350116, China

Abstract

When there is time offset, a passive position estimation method is put forward for TOA-based signal travel model from transmitter to receivers via target. By transforming the positioning model to linear equations which can be directly represented as algebraic solutions of target positions, the initial solution of target position is obtained by introducing the extra variables. Then the localization refinement is put forward to improve the performance. The simulations are conducted to test the impacts of noises on the localization accuracy. The results show that the performance of localization refinement can approximately attain the Cram-Rao lower bound of target position.

Keywords: Passive Position Estimation; Localization; TOA-based; Time Offset

1 Introduction

Driven by applications in radar and sonar technologies, crime-prevention surveillance and communications, passive localization attract numerous interests in recent years [1–3]. Due to the passive nature of the object, the passive localization problem is significantly different from the active one. The development of accurate, robust and efficient passive localization techniques has been a popular research subject over the last decade.

To improve the localization accuracy there are all kinds of proposed range-based localization methods in the literature. They include measurements of time of arrival (TOA) [4], time difference of arrival (TDOA) [5], received signal strength (RSS) [6], angle of arrival (AOA) [7]. Among these different ranging models, signal TOA measurement is relatively direct to acquire since the target can determine the signal arrival time by simply identifying and locating a known preamble from transmitted source signal. When utilizing TOA measurements for source localization, it is often assumed that the transmitter and receiver cooperate such that the signal propagation time can

∗Project supported by National Social Science Fund 10CGL005, SUES Research Project 2014-05.
*Corresponding author.
Email address: zhangj9860@sina.com (Jian ZHANG).
be found at sensor nodes. However, such collaboration between transmitter and receiver is not always available. Thus, without knowing the initial signal transmission time at the source, from TOA alone, the receiver is unable to determine the signal propagation time from its transmitter to the measuring sensor. One way to tackle this problem is to exploit the difference of pairwise TOA measurements, i.e., time-difference of arrival (TDOA). Although the dependence on the initial transmission time is eliminated by TDOA, the measurement subtraction for computing TDOA strengthens the noise and usually leads to degraded performance.

Maximum likelihood (ML) estimator can attain the Cramér-Rao lower bound (CRLB) of localization results [8]. The cost function of the ML estimator is severely nonlinear and nonconvex, so numerical solution of ML estimator strongly depends on the initialization. If the initialization is not sufficiently close to the global minimum, the numerical solution may converge to a local minimum or a saddle point causing a large estimation error. Therefore, determining an appropriate initialization point is a crucial problem in optimizing the ML cost function. As a result, some approaches have been introduced to address the shortcoming of ML problem. The semidefinite programming (SDP) by convex relaxation technique is a solution for the ML convergence problem [9,10]. In the semidefinite relaxation technique, the nonlinear and nonconvex ML problem is transformed into a convex optimization problem. The advantage of SDP technique is that its cost function does not have local minima and thus convergence to the global minimum is guaranteed. The downside is that the SDP technique is sub-optimal and cannot achieve the best possible performance in all conditions. Based on many approximations the linear analytical solutions are proposed to obtain the algebraic solutions of the target positions [11].

Based on the TOA-based signal travel model from transmitter via target to receivers, a passive position estimation method is proposed. The designed algorithm reorganizes the nonlinear equations into a set of linear equations by introducing an extra variable that is a function of the positions. To improving the localization accuracy, the localization refinement is proposed by considering the relationship between the position and the auxiliary variable. The rest of this paper is structured as follows. Section 2 presents the model specification of TOA-based passive location estimation. Section 3 derives the CRLB performance when there is time offset. Section 4 describes the algorithm design of TOA-based passive target localization. Section 5 analyzes the simulation results. The conclusion is represented in Section 6. This paper contains a number of symbols. Following the convention, we represent the matrices as bold case letters. If we denote the matrices as ($\ast$), $(\ast)^{-1}$ represents matrix inverse and $(\ast)^T$ denotes the corresponding transpose matrix. If $(\ast)$ contains noise, $\Delta(\ast)$ is the noise component.

### 2 Model Specification

For ease of exposition, we consider the passive target location estimation problem in a two-dimensional plane as shown in Fig. 1. A transmitter at location $\mathbf{x}_0 = [0 \ 0]^T$ sends out a signal that is reflected by an object at unknown location $\mathbf{x} = [x \ y]^T$. $M$ receivers at locations $\mathbf{x}_i = [x_i \ y_i]^T$, $i = 1, 2, \ldots, M$, sense direct signal from the transmitter and the reflected signal from the object that arrives a little later, for the purpose to determine the target position $\mathbf{x}$.

The model is assumed that the target reflects the signal into all directions. Using wired backbone connections between the transmitter and receivers, or high-accuracy wireless synchronization algorithms, the transmitter and receivers are synchronized. The errors of cable synchronization are negligible compared with the TOA measurement errors. Thus, at the estimation center, signal
travel times can be obtained by comparing the departure time at the transmitter and the arrival time at the receivers. The TOA from the transmitter via the target to the \(i\)th receiver is denoted as \(t_i\), which can be represented as
\[
    t_i = \frac{r_i}{c} + \tau + n_i \quad i = 1, 2, \ldots, M, \tag{1}
\]
where \(c\) is the speed of light, \(\tau\) denotes the time offset between the transmitter and the receivers, \(r_i\) represents the traveled distance from the transmitter via the target to the \(i\)-th receiver, that is,
\[
    r_i = \sqrt{x^2 + y^2} + \sqrt{(x - x_i)^2 + (y - y_i)^2} \quad i = 1, 2, \ldots, M \tag{2}
\]
In Eq. (1), \(n_i\) denotes the measurement noises which can be well modeled as a Gaussian random variable with zero mean and variance \(\delta_i^2\). Our aim is to estimate \(x\) based on the TOA measurement model depicted as Eq. (1) where the time offset \(\tau\) is assumed to be unknown.

The maximum likelihood (ML) estimator is asymptotically efficient meaning that it can achieve the CRLB accuracy when the number of measurements tends to infinity. The ML estimator of the measurement model in Eq. (1) is obtained by the following nonlinear optimization problem
\[
    \arg \min_{\Phi} \sum_{i=1}^{M} (t_i - \frac{r_i}{c} - \tau)^2, \tag{3}
\]
where \(\Phi = [x^T \quad \tau]^T\). ML solution to the localization problem can attain the position CRLB and be solved by Gauss-Newton or Levenberg-Marquardt (L-M). Based on a linear approximation to the target function Gauss-Newton or L-M method may fail when trapped in a local optimum. Even when the Gauss-Newton or L-M method converges, the solution may not be accurate because the convergence to incorrect local minimum may occur and ignoring the higher order terms in the Taylor-series expansion introduces significant error, as in the case when the measurement curves are approximately parallel. So an alternative approach to ensure global convergence is to reorganize the nonlinear equations into a set of linear equations.
3 Performance of CRLB

The CRLB matrix provides a lower bound on the covariance of any unbiased location estimator and is equal to inverse of fisher information matrix (FIM). In this section, the position CRLB of target with unknown time offset is derived. Since the time offset is not available to the estimator, it should also be taken into account as an unknown parameter. Let \( \Phi = [x \tau]^T \). The FIM is denoted as \( F \), which is also rewritten as

\[
F = -\frac{\partial^2 \ln P(t|\Phi)}{\partial \Phi^T \partial \Phi},
\]

where

\[
P(t|\Phi) = \prod_{i=1}^{M} \frac{1}{\sqrt{2\pi} \delta_i} \exp\left\{-\frac{(t_i - \frac{\tau}{c} - \tau)^2}{2\delta_i^2}\right\}
\]

Therefore the elements of matrix \( F \) can be further represented as

\[
\begin{align*}
F_{[1:2,1:2]} &= \sum_{i=1}^{M} \frac{1}{\delta_i^2} \left( \frac{x}{\sqrt{x^2+y^2}} + \frac{x-x_i}{\sqrt{(x-x_i)^2+(y-y_i)^2}} \right)^T \left( \frac{x}{\sqrt{x^2+y^2}} + \frac{x-x_i}{\sqrt{(x-x_i)^2+(y-y_i)^2}} \right) \\
F_{[1:2,3]} &= \sum_{i=1}^{M} \frac{1}{\delta_i^2} \left( \frac{x}{\sqrt{x^2+y^2}} + \frac{x-x_i}{\sqrt{(x-x_i)^2+(y-y_i)^2}} \right) \\
F_{[3,1:2]} &= F_{[1:2,3]}^T \\
F_{[3,3]} &= \sum_{i=1}^{M} \frac{1}{\delta_i^2}
\end{align*}
\]

The CRLB of the unknown parameters are the diagonal elements of the inverse of the FIM. So the CRLB of the target position \( x \) is written as

\[
\text{CRLB}(x) = F^{-1}_{[r,r]},
\]

where \( r = 1, 2 \). Given the FIM the CRLB is obtained with

\[
\text{CRLB}(x) = F^{-1}_{[1,1]} + F^{-1}_{[2,2]}
\]

4 Localization Design

In this section, we formulate the problem of passive localization as linear least square estimation and obtain the unique solutions of target positions. The localization algorithm is designed as two-step: localization primitive and refinement when the time offset is unknown. The primitive process obtains the initial solutions of the target positions. Based the coarse solutions of the primitive, the refined position is added to give an improved version of the solutions with localization refinement.

4.1 Localization primitive

Eq. (1) can be rewritten as

\[
c(t_i - \tau - n_i) - \sqrt{x^2 + y^2} = \sqrt{(x - x_i)^2 + (y - y_i)^2} \quad i = 1, 2, \ldots, M
\]
Let \( R = \sqrt{x^2 + y^2} \). Squaring both sides of Eq. (9) and ignoring the second order term, we obtain that

\[
2x_i x + 2y_i y - 2ct_i R - 2c^2 t_i \tau + c^2 \tau^2 + 2cR\tau = x_i^2 + y_i^2 - c^2 t_i^2 + 2(ct_i - c\tau - R)n_i,
\]

(10)

where \( i = 1, 2, \ldots, M \). Then the matrix solution equation constructed by stacking Eq. (10) is

\[
A \mathbf{z} = \mathbf{b} + \alpha,
\]

(11)

where \( \mathbf{z} = [x \ y \ R \ \tau \ \tau^2 \ R\tau]^T \). The row vectors of \( A \), \( \mathbf{b} \) and \( \alpha \) are respectively equal to \([2x_i \ 2y_i - 2ct_i - 2c^2 t_i \ c^2 \ 2c], [x_i^2 + y_i^2 - c^2 t_i^2] \) and \([2(ct_i - c\tau - R)n_i]\). Clearly, the matrix \( A \) is under ill-condition and its last two columns are linearly dependant of the all-one vector \( \mathbf{1}_M \). Directly solving the unknown vector \( \mathbf{z} \) from Eq. (11) is therefore impossible. However, we notice there is an orthogonal projection property of the all-one vector \( \mathbf{1}_M \) as \( \mathbf{P}\mathbf{1}_M = \mathbf{0}_M \), where \( \mathbf{P} = \mathbf{I}_M - \frac{1}{M} \mathbf{1}_M^T \mathbf{1}_M \). Multiplying this orthogonal projection matrix \( \mathbf{P} \) on both sides of Eq. (11), we can obtain that

\[
\mathbf{P}A_i \mathbf{u} = \mathbf{Pb} + \mathbf{Pa},
\]

(12)

where \( \mathbf{u} = [x \ y \ R \ \tau]^T \), the row vector of \( A_i \) is equal to \([2x_i \ 2y_i - 2ct_i - 2c^2 t_i]\). So the weighted least squares (WLS) solution to Eq. (12) is

\[
\mathbf{u} = (A_i^T \mathbf{P}^T \mathbf{W} \mathbf{P} A_i)^{-1} A_i^T \mathbf{P}^T \mathbf{W} \mathbf{b},
\]

(13)

where superscript \( \dagger \) denotes the pseudo-inverse operation, \( \mathbf{W} \) is the weighting matrix given by

\[
\mathbf{W} = E(P^T \alpha^T \alpha P)^{-1} = (P^T \Sigma P)^{-1}
\]

(14)

\[
\Sigma = E(\alpha^T \alpha) = \text{diag}\{4(ct_i - c\tau - R)^2 \delta_i\}
\]

(15)

Since the weighting matrix \( \mathbf{W} \) relies on the estimate \( R \) and \( \tau \) which are not available. We preliminarily consider \( \mathbf{W} \) as unit matrix \( \mathbf{I}_M \). Then putting the estimated initial into Eq. (14) gives an approximated \( \mathbf{Q} \) and reusing it in Eq. (13) would produce a better solution of the vector \( \mathbf{u} \). Extracting from \( \mathbf{u} \) we obtain the initial estimate \( \mathbf{x}^e \), the corresponding \( R^e \) and \( \tau^e \).

We notice that the multiplication operation in Eq. (12) is not equivalent to the operation of directly removing the last two columns of matrix \( A \) and use information is missing with the process of the multiplication operation. In the above calculations, \( \mathbf{u} \) contains three components \( x, y \) and \( R \). They were previously assumed to be independent. However \( x \) and \( y \) are clearly not independent of \( R \). The elements \( x, y \) and \( R \) have the relation equation \( R = \sqrt{x^2 + y^2} \), so the initial estimates can be further refined for improving the localization performance.

### 4.2 Localization refinement

The refinement method is based on the preliminary estimates in Eq. (13). Useful information is missing without considering relation of the elements in \( \mathbf{u} \), so the performance of localization accuracy is not optimal. Let \( \mathbf{x} = \mathbf{x}^e + \Delta \mathbf{x} \) and \( \tau = \tau^e + \Delta \tau \), where \( \Delta \mathbf{x} \) and \( \Delta \tau \) are refined errors which are assumed to be sufficiently small. Then substituting it in Eq. (10) we obtain the following expression

\[
2[x_i + \frac{c(\tau^e - t_i)x^e}{R^e}]\Delta x + 2[y_i + \frac{c(\tau^e - t_i)y^e}{R^e}]\Delta y + 2(c^2 \tau^e - c^2 t_i + cR^e)\Delta \tau
\]

\[
= (x^e - x_i)^2 + (y^e - y_i)^2 - [c(t_i - \tau^e - n_i) - R^e]^2 + 2(ct_i - c\tau^e - R^e)n_i
\]

(16)
The global matrix form of Eq. (16) can be written as

\[ C \Delta \eta = d + \alpha, \]  

(17)

where \( \Delta \eta = \begin{bmatrix} \Delta x \\
\Delta y \\
\Delta \tau \end{bmatrix}^T \), the row vectors of \( C \), \( d \) and \( \alpha \) are respectively equal to \( 2[x_i + \frac{\tau_i - \tau^e}{R_e}] \), \( 2[y_i + \frac{\tau_i - \tau^e}{R_e}] \), \( 2(c^2 \tau^e - c^2 t_i + c R^e) \), \( [(x^e - x_i)^2 + (y^e - y_i)^2 - [c(t_i - \tau^e - n_i) - R^e]^2] \) and \( 2(c(t_i - \tau^e - R^e)n_i) \). Then the weighted least squares solution to Eq. (17) is

\[ \Delta \eta = (C^T \Sigma^{-1} C)^{-1} C^T \Sigma^{-1} d \]  

(18)

The covariance of estimated \( \Delta \eta \) is denoted as \( Q \), which can also be written as

\[ Q = (C^T \Sigma^{-1} C)^{-1} \]  

(19)

Extracting from \( \Delta \eta \) to obtain the incremental part \( \Delta x \), which is added to \( x^e \) for giving a refined position vector \( x \).

\[ x = x^e + \Delta x, \]  

(20)

where the final estimation \( x \) is obtained with localization refinement.

5 Evaluation

When the time offset is unavailable, a two-step localization algorithm is designed for TOA-based travel model from transmitter via target to receivers. The designed algorithm converts the nonlinear equations to linear least square problem. The solutions of target positions are derived to obtain the initial estimates by the global expressions of the optimization equations. Then the added vectors improve the estimates by localization refinement. Computer simulations were conducted to evaluate the performance of the proposed localization algorithms.

To test the performance of localization refinement, we conduct a group of simulations with 5 receivers deployed in a 100 m x 100 m square region and the transmitter are located at (0, 0). To keep the global convergence, 5 receivers are fixed and located at (20, 50), (60, 95), (65, 10), (10, 80) and (10, 60) in the region and the target is located at (50, 50). Time offset \( \tau \) is set to 20 ns. The performance measure is MSE(\( x \)) = \( \sum_{t=1}^{T} \| x - x^o \|^2 / T \), where \( x^o \) is true node position, \( x \) is estimated node position at ensemble \( t \) and \( T = 1000 \) is the number of ensemble runs.

5.1 Impact of noises

When the noises \( \delta^2 \) are varied from 1$^2$ to 10$^2$, Fig. 2(a) plots the MSE in log-scale performance of different conditions. It can be seen from Fig. 2(a) that the position MSE in log-scale is approximately linear with the noises \( \delta^2 \) in log-scale. The position MSE of target increases with larger noises \( \delta^2 \). For instance, when all noises \( \delta^2 \) in log-scale are set to 0dB, the position MSE in log-scale of localization refinement is about -10.2dB. When the all noises \( \delta^2 \) in log-scale are all set to 20dB, the position MSE in log-scale of localization refinement is about 10.2dB. Compared with the performance of primitive, the localization refinement can attain the position CRLB which provides optimal localization accuracy.

To further show the performance of the proposed algorithms, we plot the cumulative distribution function (CDF) in Fig. 2(b) when \( \delta^2 = 1^2 \). It can be seen from Fig. 2(b) that the proposed
5.2 Impact of time offset

We also investigate the impact of time offset $\tau$ knowledge on the MSE performance of the proposed algorithms. Similarly the receivers and target are set as above and the noise $\delta^2$ is set to $1^2$. When time offset $\tau$ is varied from 10ns to 100ns, the MSE performance is plotted in Fig. 3. When the time offset is varied from 10ns to 100ns, the MSE in log-scale of localization primitive is added from -9.9dB to -2.3dB. However the MSE performance of refinement is unchangeable with the increasing of time offset. When the time offset is 10ns, the MSE in log scale of localization primitive is -10.1dB. When the time offset is increased to 100ns, the MSE in log scale of localization primitive is -9.7dB.
6 Conclusion

When there is time offset, the TOA-based passive position estimation is put forward in this paper. We transform the position estimation problem to linear equations and obtain the initial solutions of target position. Then the localization refinement is proposed for TOA-based wireless localization. In the localization primitive stage, the preliminary positions of target are obtained by introducing an extra variable. The proposed algorithms are algebraic closed-form solutions and does not require iteration or initialization. Compared with the performance of primitive, the performance of refinement method is very close to the position CRLB which provides optimal localization accuracy.

References