Channel Estimation on 60GHz Wireless Communication System Based on Subspace Pursuit

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Abstract

Due to the channel with characteristic of sparse multi-path in the 60GHz wireless communication system, the channel estimation problem can be attributed to that of sparse signals recovery. And with the consideration of the subspace pursuit (SP) algorithm is superior to the orthogonal matching pursuit (OMP) at reconstruction precision, the channel estimation technique based on the SP algorithm is presented in the 60GHz wireless communication system. First, design the OFDM multi-carrier modulation communication system. Then, establish the channel estimation mathematical model with indoor Line-of-sight. Finally, complete the reconstruction of sparse signals using SP algorithm. The experimental results and comparison analysis show that the presented technique based on the SP algorithm provides better channel estimation performances in the same pilot conditions, and it is superior to the technique based on OMP algorithm and the technique based on least square (LS) algorithm.

Keywords: Channel Estimation; 60GHz Wireless Communication System; Subspace Pursuit Algorithm; Orthogonal Matching Pursuit Algorithm

1 Introduction

The channel estimation is an important basis for channel equalization and signal detection technology in wireless communication. Therefore, the channel estimation has become an important research topic [1]. In recent years, the compressed sensing (CS) theory [2, 3] in the field of signal processing breaks the limitation of the Nyquist sampling theorem, it can sample signals at low sampling rate in accordance with the structure characteristics of signals and can effectively complete reconstruction of sparse signals and improving efficiency. Now the CS theory is applied in different systems for sparse channel estimation, such as ultra-wideband (UWB) systems [4] and underwater acoustic communication system [5], etc.

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The 60GHz wireless communication system has become an important part of the fourth generation communication since it can provide a data transfer rate with several Gbps. For there is a free frequency interval with 7GHz band in the system [6, 7], the wireless channel shows diffuse multi-path characteristics in transmission [7]. Since there is only a few non-zero taps in 60GHz wireless channel, the traditional LS algorithm [8-10] cannot accurately interpolate channel response without making full use of the sparse priori knowledge of the channel. Therefore, this method attributes the channel estimation problem as the reconstruction of sparse signal by exploiting the sparse characteristics of the channel with CS theory [9]. It can accomplish sparse channel estimation effectively by using a very limited pilot without getting the impulse response of the sub-carriers by interpolation method, which can reduce the error of channel estimation and improve spectrum efficiency [10].

In addition, the greedy algorithm is mainly used among numerous reconstruction algorithm of the CS theory. A typical class of the greedy algorithm is matching pursuit (MP) and its derivative algorithms, such as orthogonal matching pursuit (OMP), etc [11]. However, the disadvantage of these algorithm is that it still didn’t get the support in theory and the reconstruction quality is not high. Therefore, a special greedy algorithm-subspace pursuit algorithm (SP) was proposed [12]. The reconstruction accuracy of SP is higher than OMP, and the theoretical proof is abundant.

2 Channel Estimation of the 60 GHZ System

2.1 The channel model

We use the channel model proposed by the IEEE 802.15.3c group (IEEE TG3c) for living environment communication [6]. The model has obvious direct path components under the condition of line-of-sight (LOS) and directional antenna. The channel can be shown as follows.

$$h(t) = \beta \delta(t) + \sum_{l=0}^{L-1} \sum_{m=0}^{M_l-1} \alpha_{l,m} \delta(t - T_l - \tau_{l,m}) \delta(\phi - \Psi_l - \psi_{l,m})$$ (1)

$$|\alpha_{l,m}|^2 = \Omega_0 e^{-T_l / T} e^{-\tau_{l,m} / \gamma - k[1 - \delta(m)]} \sqrt{G_r(0, \Psi_l + \psi_{l,m})}$$ (2)

Where, \(t\) is the time (ns), \(\delta(\cdot)\) is the Dirac delta function, \(\beta \delta(t)\) is the direct path component, \(L\) is the number of clusters, \(m\) is the number of the arriving multipath components of the \(l\) cluster, \(M_l\) is the total number of the arriving multipath components of the \(l\) cluster, \(T_l\) is the arrival time of the first multipath component of the \(l\) cluster, \(\tau_{l,m}\) is the arrival time delay of the \(m\) multipath component with the \(l\) cluster relative to the first multipath component, \(\Omega_0\) is the average power of the first multipath of the direct-path component, \(\Psi_l\) is the arrival angle of the first multipath of the direct-path component, \(\psi_{l,m}\) is the angle of the \(m\) multipath component with the direct-path component relative to the arrival angle of the first multipath component. In the formula (2), the angle \(\alpha_{l,m}\) follows uniform distribution.

In this paper, the channel configuration and simulation parameters of the channel, i.e., CM1.1 proposed by IEEE 802.15.3c Working Group is shown in Table 1 and Table 2.

Fig. 1 shows the schematic diagram of the channel’s impulse response, where the multipath of the smaller impulse response is negligible. It can be seen that the 60GHz channel communication system is typically sparse channels.
Table 1: Channel configuration of CM1.1 TSV channel models

<table>
<thead>
<tr>
<th>Channel Model</th>
<th>Environment</th>
<th>Antenna model</th>
<th>Rx antenna HPBW</th>
<th>Sample rate</th>
</tr>
</thead>
<tbody>
<tr>
<td>CM1.1</td>
<td>Residential LOS TSV</td>
<td>Gaussian-Distributed Antenna Model</td>
<td>30 (Deg)</td>
<td>1 (GHz)</td>
</tr>
</tbody>
</table>

Table 2: Channel parameters for CM1.1 TSV channel models

<table>
<thead>
<tr>
<th></th>
<th></th>
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<th></th>
<th></th>
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<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>CM1.1</td>
<td>0.191</td>
<td>1.22</td>
<td>4.46</td>
<td>6.25</td>
<td>6.28</td>
<td>13.0</td>
<td>49.8</td>
<td>-88.7</td>
<td>2</td>
<td>0</td>
<td></td>
</tr>
</tbody>
</table>

2.2 Channel estimation model

Through the channel of 60GHz system, the output of signal is:

\[ y(n) = h^T s(n) + w(n) \]  \hspace{1cm} (3)

Where, \( s(n) = [s(n), s(n - 1), \cdots, s(n - L)]^T \) is the training sequence vector, \( s(n) \) is the pilot sequence of transmitting end, \( w(n) \) represents the Gaussian white noise which is independent and identical distribution with the input signal, \( y(n) \) is the measurement vector. The matrix form is:

\[ y = Sh + W \]  \hspace{1cm} (4)

\[ S = \begin{bmatrix} s(n) & s(n - 1) & \cdots & s(n - M) \\ s(n - 1) & s(n - 2) & \cdots & s(n - M - 1) \\ \vdots & \vdots & \ddots & \vdots \\ s(n - L) & s(n - L - 1) & \cdots & s(n - M - L) \end{bmatrix} \]  \hspace{1cm} (5)

In above formula, \( S \) is a training sequence matrix of \( L \times M \), \( W \) is the noise vector.

Our aim is to obtain \( h \) by the measurement vector \( y \) and the matrix \( s \). Since \( h \) is sparse.

There are some characteristics such as high data transmission capacity, efficient spectrum efficiency and resistance to multipath interference in the orthogonal frequency division multiplexing (OFDM) system [7]. The diagram of the OFDM system is provided in Fig. 2, where, \( x_g(n) \) is the \( s \) in the formula (3).

3 Method and Principle

3.1 Compressed sensing theory

The CS theory suggests that the original signal can reconstruct from a small amount of projection with high probability as long as the signal is sparse or presents sparse features in a transform domain. The research is the problem of solving the underdetermined equations as follows [2, 3]:

\[ y = \Phi x \]  \hspace{1cm} (6)
Here, $x$ is the $N$-dimensional data vector, $y$ is the $M$-dimensional measurement vector, $\Phi$ is the measurement matrix of $M \times N$.

And if the signal itself is not sparse then there must exist a set of transformation base $\Psi \in \mathbb{R}^{M \times M}$ which makes the projection of $x$ based on the transformation base is sparse, i.e.

$$x = \Psi \theta$$  \hspace{1cm} (7)

Where $\theta$ is $K$-sparse. Thus the measurement process can be expressed as:

$$y = \Phi x = \Phi \Psi \theta = \Theta \theta$$  \hspace{1cm} (8)

Where $\Theta = \Phi \Psi$ is sensing matrix. When $\Theta(N \times M)$ satisfies the Restricted Isometry Property (RIP), $\theta$ can be reconstructed accurately by solving the minimum 0 norm \cite{9}, i.e.,

$$\hat{\theta} = \arg \min \| \theta \|_0$$  \hspace{1cm} \text{s.t.} \\\\Theta \theta = y$$  \hspace{1cm} (9)

The purpose of reconstructing signal is obtained sparse representation $\theta$ by the formula (8) under the condition of known. It is NP-hard problem. However, the definite solution can be identified as long as finding the position of non-zero elements in $\theta$. 
3.2 The subspace pursuit algorithm

The biggest problem of signal reconstruction is to find the \( K \) columns from \( \Phi \). OMP is a greedy algorithm, which starts with an empty list, identifies one candidate during the each iteration, and adds them to the already existing list. Once a coordinate is deemed to be reliable and is added to the list, it is not removed from it until the algorithm terminates [11]. While the subspace pursuit algorithm (SP) is the method used for finding the \( K \) columns that span the correct subspace: The algorithm maintains a list of \( K \) columns from \( \Phi \), performs a simple test in the spanned space, and then refines the list. If \( y \) does not lie in the current estimate for the correct spanning space, one refines the estimate by retaining reliable candidates, discarding the unreliable ones while adding the same number of new candidates. As a consequence, the SP reevaluating the reliability of all candidates [12]. The main steps of the SP are summarized below.

**Step 1** Input: \( K, \Theta, y \). Support set: \( \hat{T}=(K \text{ indices corresponding to the largest magnitude entries in the vector } \Theta^*y) \), The residue vector: \( y_r = resid(y, \Theta_T) \), Accuracy control: \( e \), Maximum iterations: \( n \), Iterations: \( t \), Initialization \( t=1 \).

**Step 2** Iteration: At the \( t \)-th iteration, go through the following steps.

1. Merge the subscript set \( \hat{T} \) to the Support set. \( T' = \hat{T} \cup K \text{ indices corresponding to the largest magnitude entries in the vector } \Theta^*y_r \).
2. Calculate \( \theta_{p'} \) restricted to \( \Theta_{T'} \): Set \( \theta_{p'} = \Theta_{T'}^+y \). Where, \( \Theta_{T'}^+ = (\Theta_{T'}^*\Theta_{T'})^{-1}\Theta_{T'}^* \).
3. Update the support set as \( \hat{T} \): \( \hat{T} = (K \text{ indices corresponding to the largest magnitude elements of } \theta_{p'}') \).
4. Update the residue vector \( \tilde{y}_r \): \( \tilde{y}_r = resid(y, \Theta_{\hat{T}}) \).

**Step 3** Judge: If \( \|\tilde{y}_r\| \geq \|y_r\| \) or \( \|\tilde{y}_r\| \geq e \) or \( t > n \), quit the iteration, go to Step 4. Else, go to Step 2.

**Step 4** Output: The estimated signal \( \hat{\theta}, \theta_{\hat{T}} = \Theta_{\hat{T}}^+y \).

3.3 Channel estimation method based on SP algorithm

The steps of Channel estimation method based on the SP are listed as follows.

1. Design the OFDM communication system. We adopt the OFDM multi-carrier modulation.
2. Establish the mathematical model of channel estimation. Establish the mathematical model according to the channel model of 60GHz.
3. Achieve reconstruction of the sparse signal based on the SP algorithm. After the sender signals \( s(n) \) passed the channel, the received signal was obtained on the receiving end.

4 The Simulation Experiment and Comparative Analysis

In order to test the superiority of the SP algorithm, simulation experiments were conducted. The simulation experiment is conducted in MATLAB R2010b environment. In the simulation,
we adopt 4PSK modulation mode in the OFDM system, the total subcarrier number $N=864$, cyclic prefix $CP=10$. To combat the noise, repeat 100 times for each algorithm. We put the average value of the estimated results as the channel estimation result.

For comparison, we adopt the mean square error (MSE) and bit error rate (BER) to compare channel estimation performance. The equation for the mean square error is as follows.

$$MSE = \frac{E[\sum_{k} |h(k) - \hat{h}(k)|^2]}{E[\sum_{k} |h(k)|^2]}$$  

(10)

In the simulation experiments, we compare the MSE and BER of the above three kinds of algorithms at different SNR (Signal to Noise Ratio) and pilot number $P=216$, $P=144$, $P=72$ respectively. The pilot is inserted in uniform distribution in the transmission symbol. The simulation results are shown in Fig. 3, Fig. 4 and Fig. 5. Fig. 3 shows the channel estimation results based on the SP algorithm when the pilot number is 72. Fig. 4 and Fig. 5 are the MSE and BER curve of the SP, OMP and the LS algorithm in different SNR and pilot number respectively. The precise value of MSE and BER is shown in Table 3 and Table 4 respectively.

Fig. 3: The impulse response of original signal and recovered signal based on SP algorithm

The simulation results, shown in Fig. 3, indicate that there is a good reconstruction performance of the SP algorithm. Obviously shown in Fig. 4 and Fig. 5, no matter how much the pilot is, the estimation performance of SP and OMP algorithm are far higher than the LS algorithm. Meanwhile, the estimation performance of the SP algorithm is better than the OMP. Even when the pilot number is small, the BER of SP algorithm can achieve zero in a certain SNR. As the Table 4 indicates, all of the three BER values of the SP algorithm reach 0 when the SNR is 15dB, 20dB and 25dB under the pilot $P=216$, $P=144$, $P=72$ respectively, which gets a good estimated performance. The two BER values of OMP algorithm reach 0 when the SNR is 15dB and 25dB under the pilot $P=216$, $P=144$ respectively, while the BER value of OMP algorithm is small when the SNR is 30 dB under the pilot $P=72$. Thus, SP algorithm has higher estimation performance than OMP algorithm. And yet for the LS algorithm, even when the SNR is 30dB the BER is 0.2297Db, 0.2749dB, 0.3288dB under the pilot $P=216$, $P=144$, $P=72$ respectively, which presents a greater BER.
Shown in Fig. 4 and Fig. 5, with the increasing of the SNR, the MSE and BER of the SP and OMP algorithm decreased rapidly, while the traditional LS algorithm’s tends to a horizontal line. In Table 3 and Table 4, with the SNR increases from 0 to 30dB, the MSE of the SP algorithm rolls down from 0.4279dB to 7.357e-005dB, while the OMP algorithm rolls down from 0.698dB to 0.698dB and the LS algorithm from 2.256dB to 2.256dB. Meanwhile, the BER of the SP algorithm rolls down from 0.1406dB to 0, while the OMP algorithm rolls down from 0.2288dB to 0 and the LS algorithm from 0.2889dB to 0.

Table 3: Mean square error (MSE) values for the three algorithms

<table>
<thead>
<tr>
<th>Algorithms</th>
<th>Pilot number P</th>
<th>0dB SNR</th>
<th>5dB SNR</th>
<th>10dB SNR</th>
<th>15dB SNR</th>
<th>20dB SNR</th>
<th>25dB SNR</th>
<th>30dB SNR</th>
</tr>
</thead>
<tbody>
<tr>
<td>OMP</td>
<td>P=216</td>
<td>0.698</td>
<td>0.2097</td>
<td>0.0487</td>
<td>0.009185</td>
<td>0.001349</td>
<td>0.0004625</td>
<td>0.0001256</td>
</tr>
<tr>
<td>SP</td>
<td>P=216</td>
<td>0.4279</td>
<td>0.1038</td>
<td>0.02027</td>
<td>0.002361</td>
<td>0.00086</td>
<td>0.0002574</td>
<td>77.357e-005</td>
</tr>
<tr>
<td>LS</td>
<td>P=216</td>
<td>2.256</td>
<td>1.667</td>
<td>1.478</td>
<td>1.424</td>
<td>1.41</td>
<td>1.407</td>
<td>1.404</td>
</tr>
<tr>
<td>OMP</td>
<td>P=144</td>
<td>0.9307</td>
<td>0.319</td>
<td>0.08022</td>
<td>0.02054</td>
<td>0.003134</td>
<td>0.0005882</td>
<td>0.00002023</td>
</tr>
<tr>
<td>SP</td>
<td>P=144</td>
<td>0.6786</td>
<td>0.1905</td>
<td>0.04297</td>
<td>0.008091</td>
<td>0.001483</td>
<td>0.0003914</td>
<td>0.0003124</td>
</tr>
<tr>
<td>LS</td>
<td>P=144</td>
<td>2.746</td>
<td>2.07</td>
<td>1.881</td>
<td>1.852</td>
<td>1.806</td>
<td>1.802</td>
<td>1.797</td>
</tr>
<tr>
<td>OMP</td>
<td>P=72</td>
<td>1.233</td>
<td>0.5828</td>
<td>0.3137</td>
<td>0.1122</td>
<td>0.02743</td>
<td>0.01408</td>
<td>0.0002657</td>
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<tr>
<td>SP</td>
<td>P=72</td>
<td>1.156</td>
<td>0.4608</td>
<td>0.1544</td>
<td>0.0392</td>
<td>0.007539</td>
<td>0.001021</td>
<td>0.0003244</td>
</tr>
<tr>
<td>LS</td>
<td>P=72</td>
<td>4.528</td>
<td>4.01</td>
<td>3.813</td>
<td>3.782</td>
<td>3.747</td>
<td>3.754</td>
<td>3.753</td>
</tr>
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</table>

Table 4: Bit error rate (BER) values for the three algorithms

<table>
<thead>
<tr>
<th>Algorithms</th>
<th>Pilot number P</th>
<th>0dB SNR</th>
<th>5dB SNR</th>
<th>10dB SNR</th>
<th>15dB SNR</th>
<th>20dB SNR</th>
<th>25dB SNR</th>
<th>30dB SNR</th>
</tr>
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<tbody>
<tr>
<td>OMP</td>
<td>P=216</td>
<td>0.2288</td>
<td>0.05017</td>
<td>0.00544</td>
<td>0.0003472</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>SP</td>
<td>P=216</td>
<td>0.1406</td>
<td>0.01591</td>
<td>0.001794</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
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<tr>
<td>LS</td>
<td>P=216</td>
<td>0.2889</td>
<td>0.2584</td>
<td>0.2416</td>
<td>0.2348</td>
<td>0.231</td>
<td>0.2311</td>
<td>0.2303</td>
</tr>
<tr>
<td>OMP</td>
<td>P=144</td>
<td>0.3015</td>
<td>0.09172</td>
<td>0.01071</td>
<td>0.001562</td>
<td>5.787e-005</td>
<td>0</td>
<td>0</td>
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<tr>
<td>SP</td>
<td>P=144</td>
<td>0.2373</td>
<td>0.04508</td>
<td>0.003993</td>
<td>0.0002894</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>LS</td>
<td>P=144</td>
<td>0.3019</td>
<td>0.2822</td>
<td>0.2738</td>
<td>0.2737</td>
<td>0.2759</td>
<td>0.2742</td>
<td>0.2738</td>
</tr>
<tr>
<td>OMP</td>
<td>P=72</td>
<td>0.3417</td>
<td>0.1872</td>
<td>0.07998</td>
<td>0.01881</td>
<td>0.001968</td>
<td>0.001505</td>
<td>0.001736</td>
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<tr>
<td>SP</td>
<td>P=72</td>
<td>0.3898</td>
<td>0.1765</td>
<td>0.02922</td>
<td>0.003299</td>
<td>0.0002315</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>LS</td>
<td>P=72</td>
<td>0.3418</td>
<td>0.336</td>
<td>0.3315</td>
<td>0.3299</td>
<td>0.3278</td>
<td>0.3281</td>
<td>0.3275</td>
</tr>
</tbody>
</table>
5 Conclusions

In this paper, channel estimation of the 60GHz wireless communication system based on the SP algorithm of compressed sensing is proposed, i.e., we transform channel model of the communication system into CS solvable reconstruction model for its sparse features and accurately estimate the sparse channel by using the SP algorithm. The experimental results show that, the estimation performance based on the SP algorithm is superior to the OMP and least square (LS) algorithm and it can acquire an accurate estimation with a limited pilot number.

References


