An Adaptive Relay Assignment and Power Allocation Algorithm in CRNs

Mingyue ZHOU\textsuperscript{1,2,*}, Xiaohui ZHAO\textsuperscript{2}

\textsuperscript{1}College of Information and Technology, Jilin Normal University, Siping 136000, China
\textsuperscript{2}Key Laboratory of Information Science, School of Communication Engineering, Jilin University, Changchun 130012, China

Abstract

In this paper, we present an adaptive relay assignment and power allocation algorithm in cognitive radio networks by using cooperative relay technologies which can improve the capacity of the cognitive system and extend coverage area. This algorithm converges faster than classic algorithms. The main target of the adaptive algorithm is to maximize each user's capacity while guarantees SNR of the cognitive system. And this cognitive system will adaptive select best relay for participation in cooperative transmission. Therefore, power control problem can be formulated as a mixed-integer nonlinear programming problem solved under KKT conditions. Simulation results show that the proposed adaptive algorithm performs better and effectively compared with the classical ones.

Keywords: Relay Assignment; Power Allocation; Cognitive Radio

1 Introduction

As the rapid development of wireless communication networks, the contradiction between limited spectrum resources and rapidly increasing demand for spectrum is becoming significantly acute [1]. Through dynamic spectrum allocation and spectrum sharing, cognitive radio technology is proposed to improve spectrum efficiency. Transmit power of cognitive user has to be restricted for the reason that we must guarantee communication quality of primary user, which will reduce the coverage of cognitive network. And it is difficult to improve capacity of the cognitive system at same time. While relay transmission is an important strategy to improve power efficiency and increase system capacity. In the case of no reduction of transmit power, the cognitive network based on cooperative relay can offer significant capacity enhancement and may further augment the coverage.

Recently, the research about relay selection and power allocation in wireless communication systems has been conducted by many literatures. Muhammad Naeem etc. propose a greedy iterative

\*Project supported by the National Nature Science Foundation of China (No.61171079).
\*Corresponding author.
Email address: zmyjlu@163.com (Mingyue ZHOU).
joint multiple relay assignment and power allocation scheme for cognitive radio systems in [2]. In [3], the power and bandwidth allocation of amplify-and-forward cooperative communications is modeled as Stackelberg market framework. And a distributed scheme obtaining power and bandwidth allocations is presented. Mohammad Faisal Uddin etc. Reference [4] proposes a mixed Boolean-convex optimization problem which substantially reduces computation time and achieves near optimal solutions in order to solve joint relay assignment and power allocation for multicast cooperative networks. In [5], each subcarrier pair is assigned to all relays, and a joint power allocation and subcarrier pairing algorithm is presented and the optimization problem is solved by dual method. Yuan Liu and Meixia Tao present an optimal channel and relay assignment scheme by transforming it into a maximum weighted bipartite matching problem using graph theoretical approach [6]. Hao Zhang and Yuan Liu etc. formulate the resource allocation with subcarrier in OFDMA two-way relay networks as a mixed integer programming problem. And an efficient algorithm is presented by using dual method in [7]. Dejun Yang and Guoliang Xue investigate a relay assignment scheme without reducing system performance. By using payment mechanism, the solution to the scheme can converge to the optimal answer in [8]. Considering relay selection, power allocation and subcarrier assignment problem to a joint optimization problem, two low-complexity suboptimal algorithms are studied in [9].

In the analysis of the above literatures, we can conclude that through the cooperation between cognitive nodes, cognitive and cooperative wireless networks can get diversity gain and improve performance of systems. Cooperative power allocation scheme in cognitive radio networks need to avoid interference to primary users. We will conduct spectrum underlay for relay assignment and power allocation in cognitive radio networks. This approach requires spectrum sharing between primary and cognitive users on the condition that the interference from cognitive users must be below the threshold that primary receiver can tolerate. When cognitive users reduce transmit power, it will limit the system capacity and the coverage performance. How to improve transmission performance of the cognitive systems without increasing transmit power of cognitive users become a significant question for the study and discussion.

In this paper, we present an adaptive interference-aware relay assignment and power allocation algorithm in cognitive radio networks, which is computationally efficient algorithm with relay selection mechanism based on power optimization. This adaptive algorithm aims to maximize cognitive capacity while keeping primary user’s QoS. In addition, SNR of the cognitive receiver is lower than target SNR when signals are directly transmitted. Best relay node for corresponding user will be adaptively selected to participate in the cooperative transmission. Then the power control problem becomes a mixed-integer nonlinear programming problem. From KKT conditions, we will obtain the optimal power and enhance the capacity for the cognitive system.

2 System Model

We consider the cognitive radio networks with $I$ pairs of cognitive users and $R$ relays, i.e., there is no central control node. And there are $M$ primary users in the region of interest. We assume that each relay can not shared by different cognitive users. The signal is transmitted through a low time-varying channel so that each user is able to perfectly estimate its local channel state information.

Each cognitive user’s transmit power must be lower than its maximum power for which in
reality we should guarantee QoS requirements of cognitive users [10], i.e.,
\[ 0 \leq p_i \leq P_{i}^{\text{max}}, \forall i = 1, 2, \cdots, I \] (1)
where \( p_i \) denotes the transmit power of cognitive user \( i \). \( P_{i}^{\text{max}} \) is the maximum permissible power for cognitive user \( i \).

In order to guarantee QoS of primary user, the interference power from each cognitive user should not exceed the primary users’ permissible threshold that primary user \( m \) can tolerate [11], i.e.,
\[ p_i |e_{im}|^2 \leq I_{im}, \forall i = 1, 2, \cdots, I, \forall m = 1, 2, \cdots, M \] (2)
where \( e_{im} \) is the channel gain from transmitter \( i \) of cognitive user to receiver \( i \) of primary user \( m \). \( I_{im} \) denotes the maximum interference level of primary user \( m \) for cognitive user \( i \).

The optimization objective of the cognitive system is to maximize the total capacity of all cognitive users under the conditions of maximum transmit power constraint (1) and interference power constraint (2). The optimization problem can be depicted as follows
\[
\begin{align*}
\max_{\mathbf{p}} & \sum_{i=1}^{I} \log_2 \left( 1 + \frac{p_i |h_{ii}|^2}{\sigma_d^2} \right) \\
\text{s.t.} & \quad 0 \leq p_i \leq P_{i}^{\text{max}} \\
& \quad p_i |e_{im}|^2 \leq I_{im}
\end{align*}
\] (3)
where \( \sigma_d^2 \) is the background noise power at cognitive receiver \( i \). \( h_{ii} \) is the channel gain from transmitter \( i \) to receiver \( i \) for cognitive user \( i \).

Since the optimization problem (3) is not convex optimization problem, we can reformulate this problem as the following optimization problem
\[
\begin{align*}
\min_{\mathbf{p}} & -\sum_{i=1}^{I} \log_2 \left( 1 + \frac{p_i |h_{ii}|^2}{\sigma_d^2} \right) \\
\text{s.t.} & \quad p_i \in K_i, \quad i = 1, \cdots, I
\end{align*}
\] (4)
where \( K_i \) denotes a closed convex subset of \( \mathbb{R}^n \), and
\[
K_i = \left\{ p \in \mathbb{R}^{1 \times n} \left| p_i |e_{im}|^2 \leq I_{im}, \forall i = 1, 2, \cdots, I, \forall m = 1, 2, \cdots, M; \right. \right. \\
p_i \leq P_{i}^{\text{max}}, \forall i = 1, 2, \cdots, I \right\}
\] (5)

This is a nonlinear programming problem. We can get the optimal solution from KKT condition [12], i.e.,
\[
p_i^* = \frac{1}{\lambda + \mu |h_{ii}|^2} - \frac{\sigma_d^2}{|h_{ii}|^2}
\] (6)
\( \lambda \) and \( \mu \) are Lagrange multiplier. They will be updated by the following equation
\[
\left\{ \begin{array}{l}
\lambda (t + 1) = [\lambda (t) + \xi (p_i |e_{im}|^2 - I_{im})]^+
\\
\mu (t + 1) = [\mu (t) + \zeta (p_i - P_{i}^{\text{max}})]^+
\end{array} \right.
\] (7)
where \([z]^+ = \max (z, 0)\). \( \xi \) and \( \zeta \) denote the proper step size for this optimization problem, \( t \) and \( t \) is iteration number.
Thus the SNR at receiver $i$ of cognitive user in the cognitive radio network can be calculated by $\text{SNR}_i = \frac{p_i |h_{ii}|^2}{\sigma_d^2}$. The target SNR is marked by $\text{SNR}_i^{\text{tar}}$. Usually we hope that SNR of cognitive user is better than target SNR, that is

$$\text{SNR}_i - \text{SNR}_i^{\text{tar}} \geq 0 \quad (8)$$

Some cognitive radio systems requirement for signal-to-noise ratio is not strict; thus the proposed schemes based on (6) are enough. When we have rigorous request for SNR in cognitive radio networks, this scheme may not guarantee SNR requirement. If we want to improve the signal-to-noise ratio in this cognitive system, we need to use other methods.

We use a heterogeneous network consisting of different requirements, different networks and mutual penetration of different coverage. Relay coordination communication and relay assignment based on the actual situation is considered.

In this network, the transmit power of relay $r$ is not exceed its the maximum available power, i.e.,

$$0 \leq p_r \leq P_r^{\text{max}}, r = 1, 2, \cdots, R \quad (9)$$

where $p_r$ and $P_r^{\text{max}}$ denote the transmit power and the maximum available transmit power for relay $r$ respectively.

The cooperative power allocation strategy also needs considering the avoidance of the interference to primary users. We require that the interference power from relay $r$ does not exceed the permissible power level $T_{rm}$ for primary user $m$, i.e.,

$$p_r |E_{rm}|^2 \leq T_{rm}, \forall r = 1, 2, \cdots, R, \forall m = 1, 2, \cdots, M \quad (10)$$

where $T_{rm}$ is the predefined maximum permissible threshold and $E_{rm}$ is the channel gain between the relay $r$ and the primary user $m$.

From the point of current research for relay coordination communication, we consider a half-duplex mode where each transmission cycle is divided into two time slots. The source node broadcasts its information in first time slot, and both relay nodes and destination nodes receive the broadcast information. And the relays amplify the received signal and re-transmit the amplified signal in second time slot.

Considering the forward path channel between source node and information obtaining nodes-the channel based on relay strategy is equivalent to a single input two output channel complex Gaussian noise channel [13]. In AF mode, maximum ratio combination has the largest system capacity. The channel capacity of cognitive user with the help of relay for AF can be expressed as [14]

$$C_{ir} = \frac{1}{2} \log_2 \left(1 + \frac{p_i |h_{ii}|^2}{\sigma_d^2} + \frac{p_i |g_{ir}|^2}{p_r |G_{ri}|^2} \frac{p_r |G_{ri}|^2}{\sigma_d^2} \frac{p_r |G_{ri}|^2}{\sigma_d^2} \right), \forall i = 1, 2, \cdots, I, \forall r = 1, 2, \cdots, R \quad (11)$$

where $\sigma_d^2$ denotes the background noise at relay $r$. The positive parameter $g_{ir}$ and $G_{ri}$ are the channel gains from the cognitive transmitter $i$ to the relay $r$ and the relay $r$ to the cognitive receiver $i$ respectively.

Let $r_{id} = \frac{p_i |h_{ii}|^2}{\sigma_d^2}, r_{ir} = \frac{p_i |g_{ir}|^2}{\sigma_d^2}$, and $r_{rd} = \frac{p_r |G_{ri}|^2}{\sigma_d^2}$, thus, we have

$$C_{ir} = \frac{1}{2} \log_2 \left(1 + r_{id} + \frac{r_{ir} r_{rd}}{r_{ir} + r_{rd} + 1} \right) \quad (12)$$
Mathematically, we can formulate the optimization problem of maximizing the capacity as the following mixed-integer linear programming problem, i.e.,

$$\max \sum_{i=1}^{I} \sum_{r=1}^{R} \frac{1}{2} \log_2 \left(1 + r_{id} + \frac{r_{ir}r_{rd}}{r_{ir}+r_{rd}+1}\right)$$

subject to:

$$0 \leq p_i \leq P_i^{\max}$$
$$0 \leq p_r \leq P_r^{\max}$$
$$p_i |e_{im}|^2 \leq I_{im}$$
$$p_r |E_{rm}|^2 \leq T_{rm}$$

(13)

3 Adaptive Relay Assignment and Power Allocation Algorithm

How to assign cooperative relays and allocate power in cognitive radio networks is one of the most important issues. We should guarantee QoS of primary user and improve the capacity for the whole cognitive network.

First of all, we consider relay assignment problem in the cognitive network. It is assumed that each relay is not shared by different cognitive users. Each cognitive user chooses its best relay which has the higher channel gain. Thus the problem (13) can be represented as

$$\min - \sum_{i=1}^{I} \sum_{r=1}^{R} \frac{1}{2} \log_2 \left(1 + r_{id} + \frac{r_{ir}r_{rd}e_{ir}}{r_{ir}+r_{rd}+1}\right)$$

subject to:

$$0 \leq p_i \leq P_i^{\max}$$
$$0 \leq p_r \leq P_r^{\max}$$
$$p_i |e_{im}|^2 \leq I_{im}$$
$$\varepsilon_{ir}p_r |E_{rm}|^2 \leq T_{rm}$$
$$\sum_{r=1}^{R} \varepsilon_{ir} \leq 1$$

(14)

where $\varepsilon_{ir}$ denotes a binary assignment indicator about relay assignment. If $\varepsilon_{ir} = 1$, it means that the relay $r$th is assigned to the $i$th cognitive user. Otherwise $\varepsilon_{ir} = 0$, it indicates that the $r$th relay is not assigned to the $i$th cognitive user. And $\sum_{r=1}^{R} \varepsilon_{ir} \leq 1$ guarantees that each relay is assigned to only one cognitive user.

The objective function for the optimal problem (14) has two variables $p_i$ and $p_r$. We will decouple the transmit power $p_r$ of relay $r$ for this proposed scheme. From constraints $0 \leq p_r \leq P_r^{\max}$ and $p_r |E_{rm}|^2 \leq I_{rm}$, we can conclude that the range of $p_r$ is

$$0 \leq p_r \leq \min \left( P_r^{\max}, \frac{I_{r1}}{|E_{r1}|^2}, \frac{I_{r2}}{|E_{r2}|^2}, \ldots, \frac{I_{rM}}{|E_{rM}|^2} \right)$$

(15)

In order to enhance the capacity and improve SNR for the cognitive radio network, we maximize the transmit power of the relay $r$ under the condition of QoS guarantee of primary user. Hence the appropriate value of $p_r$ for relay $r$ is

$$p_r^{\text{opt}} = \min \left( P_r^{\max}, \frac{I_{r1}}{|E_{r1}|^2}, \frac{I_{r2}}{|E_{r2}|^2}, \ldots, \frac{I_{rM}}{|E_{rM}|^2} \right)$$

(16)
Thus the optimal problem (14) can be represented as
\[
\min - \sum_{i=1}^{I} \sum_{r=1}^{R} \frac{1}{2} \log_2 \left( 1 + r_{id} + \frac{r_{ir} \sigma_r^2}{r_{ir} + r_o \varepsilon_{ir} + 1} \right)
\]
\[
s.t. \quad \begin{cases} 0 \leq p_i \leq P_i^{\text{max}} \\ p_i |\epsilon_{im}|^2 \leq I_{im} \\ \sum_{r=1}^{R} \varepsilon_{ir} \leq 1 \end{cases}
\]
(17)
where \( r_o = \frac{\sigma_o^2 |G_{ri}|^2}{\sigma_d^2} \).

It is not difficult to find that Lagrange function of the optimization problem (17) as
\[
L(p, \theta, \vartheta) = - \sum_{i=1}^{I} \sum_{r=1}^{R} \frac{1}{2} \log_2 \left( 1 + r_{id} + \frac{r_{ir} \sigma_r^2}{r_{ir} + r_o \varepsilon_{ir} + 1} \right) + \theta \left( p_i |\epsilon_{im}|^2 - I_{im} \right) + \vartheta \left( p_i - P_i^{\text{max}} \right)
\]
(18)
where \( \theta \) and \( \vartheta \) are the vector of Lagrange dual variables for the constraints. \( \theta \) and \( \vartheta \) are updated by
\[
\begin{cases} \theta (t + 1) = [\theta (t) + \alpha \left( p_i |\epsilon_{im}|^2 - I_{im} \right)]^+ \\ \vartheta (t + 1) = [\vartheta (t) + \beta \left( p_i - P_i^{\text{max}} \right)]^+ \end{cases}
\]
(19)
where \( \alpha \) and \( \beta \) present proper step size.

When cognitive user \( i \) chooses relay \( r \), the KKT conditions for cognitive user \( i \) is \( \frac{\partial L(p, \theta, \vartheta)}{\partial p_i} = 0 \).

We have the optimal solution for problem (14) as follows
\[
p_i^{\text{opt}} = \frac{1}{\sigma_d^2 |g_{ir}|^2} \left\{ \frac{1}{2 \left( \theta |\epsilon_{im}|^2 + \vartheta \right)} \frac{|h_{ir}|^2 s_i^2 + |g_{ir}|^2 p_r^{\text{opt}} |G_{ri}|^2 u_i}{s_i^2 \left( 1 + p_i \frac{|h_{ir}|^2}{\sigma_d^2} + p_i |g_{ir}|^2 p_r^{\text{opt}} |G_{ri}|^2 \right) - u_i} \right\}
\]
(20)
where \( s_i = p_i \sigma_d^2 |g_{ir}|^2 + p_r^{\text{opt}} \sigma_r^2 |G_{ri}|^2 + \sigma_d^2 \sigma_r^2 \) and \( u_i = p_r^{\text{opt}} \sigma_r^2 |G_{ri}|^2 + \sigma_d^2 \sigma_r^2 \).

The detailed steps for the proposed adaptive relay selection and power allocation algorithm in cognitive radio networks are summarized as follows

S1: Initialization: Set \( t = 0, \lambda > 0, \mu > 0, p_i = 0 \).

S2: Update: Update \( \lambda \) and \( \mu \) by (7) respectively.

S3: Calculation: Get the optimal power \( p_i^{\text{opt}} \) in cognitive network without relay communication by (6) and go to S2.

S4: Comparison: Calculate the value of inequality (8). If the value of inequality (8) is not true, go to next step S5, else, end the execution of this program. When SNR for the cognitive radio network without relay communication satisfies the requirement, the relay cooperative communication will not be adopted and vice versa.

S5: Re-initialization: \( \theta > 0, \vartheta > 0, p_i > 0 \).

S6: Determination: The cognitive user \( i \) chooses its best relay with higher channel gain (\( \varepsilon_{ir} = 1 \)). The rest of relay number is \( R = R - 1 \).

S7: Calculation: Calculate \( p_i^{\text{opt}} \) by equation (16).

S8: Update: Update \( \theta \) and \( \vartheta \) by (19) respectively.

S9: Calculation: Get the optimal transmit power \( p_i^{\text{opt}} \) of cognitive user \( i \) by (16). Go to S8.
4 Numerical Simulation Results

Numerical results are presented to verify the performance of our proposed adaptive relay selection and power allocation algorithm in cognitive radio networks. Suppose that there are five cognitive users and one primary user randomly locating around the region of interest with eight relays in the network. For simplicity, the maximum transmit power all relay nodes is equal to 2.5. The maximum transmit power of cognitive users is $P_{\text{max}} = [2, 2.36, 2.2, 2.08, 2.15]$. The background noise power at cognitive receiver $i$ and relay $r$ is randomly chosen from the range of $[0.1, 0.25]$. The average channel gain of its communication links is $E[h_{ii}] = 0.8$ for cognitive users. The average channel gain from cognitive transmitters to the relays and the relays to the cognitive receivers are assumed to be $E[g_{ir}] = 2$ and $E[G_{ri}] = 0.9$ respectively. The average channel gain from cognitive users to primary user and the relays to the primary user is both supposed to be 0.4.

The performance of the classic power allocation algorithm in cognitive radio network is presented in Fig. 1 to Fig. 2. Fig. 1 shows the transmit power and the SNR of each cognitive user for classic algorithm. From the perspective of game theory, the classic power allocation algorithm of the cognitive system has reached the state of Nash equilibrium, which is the necessary condition to reach Pareto Optimality.

The solid line in Fig. 1 about SNR is the target SNR threshold of this system and it is equal to 10dB. It is observed that there are three users get satisfactory QoS when the system arrives at equilibrium point. The transmit power for cognitive user 2 and cognitive user 5 are more than those of the other three cognitive users, but they do not obtain satisfactory results. Even the communication of our correspondence with the cognitive 2 and cognitive 5 may be interrupted.

![Fig. 1: Transmit power and SNR for classic algorithm](image)

Fig. 2 presents the capacity of each cognitive user for classic algorithm. The capacity of cognitive user 2 and 5 is less than that of others too.

To get better QoS, we use cooperative relay technologies to improve signal-to-noise ratio, spectrum efficiency, and capacity, and to extend coverage area in cognitive radio networks. We call our proposed scheme as adaptive algorithm for short.

Fig. 3 presents the transmit power and SNR about cognitive user 2 and cognitive user 5 with two power allocation algorithms. It is observed that the transmit power of cognitive user 2 for classic algorithm is far less than that of the corresponding user for adaptive algorithm. The transmit power of cognitive user 5 for two methods have nearly equal power. However, the SNR
for both users is greatly improved in adaptive algorithm.

The result of analysis of the SNR for two cognitive users by two algorithms is shown that the SNR for adaptive algorithm is far more than that of the classic algorithm, and also is better than the target SNR threshold. Comparing with the classic algorithm, the adaptive method can greatly improve the SNR and system capacity. Furthermore the convergence speed of this algorithm is faster than that of the classic method. It is obvious that the adaptive algorithm has better performance than classic algorithm.

Fig. 3: Transmit power comparator and SNR comparator

Fig. 4 shows power allocation and SNR of each cognitive user for adaptive algorithm. It is demonstrated that the adaptive relay selection and power allocation strategy can quickly reach Nash equilibrium. Compared with Fig. 1, the total transmit power of all cognitive users is less than that of the classic method and it can satisfy the SNR requirement of the cognitive system. It is also shown that the SNR of each cognitive user is larger than the target SNR and each user gets satisfactory QoS.

The capacity of each cognitive user and the interference power at primary user generated by each cognitive user are shown in Fig. 5. Simulation results show that power allocation using adaptive algorithm has high spectral efficiency. And channel capacity is increased effectively. The adaptive algorithm reduces power consumption with almost maximum capacity while achieve Nash equilibrium.

The interference power at primary user generated by each cognitive user is shown in Fig. 5 about interference power. The solid line is the interference power threshold which the primary
user can tolerate. It is shown that the interference power from each user is below the threshold. The normal communication does not affect the adaptive algorithm.

5 Conclusion

Based on relay cooperation in cognitive radio networks, an adaptive relay selection and power allocation scheme has been proposed. The proposed algorithm can greatly improve system SNR and capacity with fast convergence speed. Numerical results have shown that the algorithm provides better performance and ensure system seamless communication in comparison with classical power allocation algorithms.

References


[5] Xueyi Li, Qi Zhang, Guangchi Zhang, Jiayin Qin, Joint power allocation and subcarrier pairing for cooperative OFDM AF Multi-Relay Networks, IEEE Communications Letters, May 2013, 17(5): 872-875.


