Robust Delay-dependent $H_\infty$ Filter for T-S Fuzzy Descriptor Time-delay Systems with Parameter Uncertainties

Yuechao MA, Huijie YAN*

College of Science, Yanshan University, Qinhuangdao 066004, China

Abstract

This paper deals with the delay-dependent robust $H_\infty$ filter for a class of Takagi-Sugeno (T-S) fuzzy descriptor time-delay systems with parameter uncertainties. The uncertainties in the system are time-varying and bounded. By simplifying the T-S fuzzy descriptor system to the time-delay system, and base on Lyapunov stability theory in time-delay system, a sufficient condition on the existence of robust $H_\infty$ filters is derived. The design problem of robust $H_\infty$ filter is turned into the solution problem of linear matrix inequalities. And then, the delay-dependent robust $H_\infty$ filtering problem of T-S fuzzy descriptor time-delay systems can be solved in terms of a robust $H_\infty$ filtering problem without uncertainties. Finally, the example is provided to demonstrate the effectiveness of the proposed method.

Keywords: Takagi-Sugeno Fuzzy Descriptor Time-delay System; Takagi-Sugeno Fuzzy System; Robust $H_\infty$ Filter; Fuzzy System Model

1 Introduction

In the past decades, the method of filtering is generally being used, and it has attracted great interests. Tanguchi spread the normal T-S fuzzy model into a more generalized condition, and put forward T-S fuzzy descriptor system model in [1], which has developed a new way for the investigation of non-linear descriptor system and time-varying system (see, for example, [2–4]). Robust fuzzy Lyapunov stabilization for uncertain and disturbed T-S descriptors is studied in [5]. Delay-dependent stability and $H_\infty$ control for a class of fuzzy descriptor systems with time-delay are studied in [6]. A descriptor system approach to uncertain fuzzy control system design via fuzzy Lyapunov functions is discussed in [7]. The passivity control for a kind of T-S fuzzy descriptor system is studied in [8]. $H_\infty$ filtering of time-delay T-S fuzzy systems based on piecewise Lyapunov-Krasovskii functional is studied in [9].
Descriptor systems are often used in the network, economy, robotics and other fields, and it has received extensive attention of people, so the filtering of descriptor system is especially important. Reduced order $H_\infty$ filtering for descriptor systems is studied in [10]. Delay-dependent $H_\infty$ filter for descriptor Markovian jump time-delay systems is studied in [11]. Delay dependent robust $H_\infty$ filter for uncertain discrete-time descriptor systems with interval time-varying delay is studied in [12]. The $H_\infty$ filter for descriptor systems with communication delays is studied in [13]. Delay dependent robust filter for descriptor systems with time-varying matrix, and is assumed to be of the following form:

\[ E \dot{x}(t) = (A_{i1} + \Delta A_{i1})x(t) + (A_{i2} + \Delta A_{i2})x(t - d_i(t)) + (B_i + \Delta B_i)w(t) \]

\[ y(t) = (C_{i1} + \Delta C_{i1})x(t) + (C_{i2} + \Delta C_{i2})x(t - d_i(t)) + (D_i + \Delta D_i)w(t) \]

\[ z(t) = (L_i + \Delta L_i)x(t), \quad i = 1, 2, \cdots, r, \]

Where $M_{ij}$ denote the fuzzy sets; $r$ denotes the number of model rules; $x(t) \in R^n$ is the input vector; $w(t) \in R^m$ is the disturbance vector of the system which belongs to $L_2[0, \infty)$; $y(t) \in R^q$ is the measurable output vector; $z(t) \in R^p$ is the signal vector to be estimated; The matrix $E \in R^{n \times n}$ may be singular and assume that rank$E = r \leq n$; $A_{i1}, A_{i2}, B_i, C_{i1}, C_{i2}, D_i, L_i$ are known real constant matrices with appropriate dimensions; $\xi_1(t)$, $\xi_2(t)$, $\cdots$, $\xi_p(t)$ are the functions of state variables; $d_i(t)$ is a time-varying continuous function that satisfies $0 \leq d_1 \leq d_2, d_1(t) \leq \mu; \Delta A_{i1}, \Delta A_{i2}, \Delta B_i, \Delta C_{i1}, \Delta C_{i2}, \Delta D_i, \Delta L_i$ are unknown matrices describing the model uncertainties, and are assumed to be of the form:

\[
\begin{bmatrix}
\Delta A_{i1} & \Delta A_{i2} & \Delta B_i \\
\Delta C_{i1} & \Delta C_{i2} & \Delta D_i
\end{bmatrix}
= \begin{bmatrix}
H_{1i} \\
H_{2i}
\end{bmatrix}
F_i \begin{bmatrix}
E_{1i} & E_{2i} & E_{3i}
\end{bmatrix},
\Delta L_i = H_{3i}F_iE_{4i}.
\]

Where $H_{1i}, H_{2i}, H_{3i}, E_{1i}, E_{2i}, E_{3i}, E_{4i}$ are known real constant matrices. $F_i$ is unknown real time-varying matrix, and is assumed to be of the following form:

\[ F_i^T F_i \leq I. \]

The parametric uncertainties $\Delta A_{i1}, \Delta A_{i2}, \Delta B_i, \Delta C_{i1}, \Delta C_{i2}, \Delta D_i, \Delta L_i$ are said to be admissible if both (2) and (3) hold.

## 2 Problem Description

Consider the following uncertain T-S fuzzy descriptor time-delay model, which is described by plant Rule $R_i$: if $\xi_1(t)$ is $M_{1i}$ and $\xi_2(t)$ is $M_{2i}$, $\cdots$, $\xi_p(t)$ is $M_{pi}$, then

\[ E \dot{x}(t) = (A_{i1} + \Delta A_{i1})x(t) + (A_{i2} + \Delta A_{i2})x(t - d_i(t)) + (B_i + \Delta B_i)w(t) \]

\[ y(t) = (C_{i1} + \Delta C_{i1})x(t) + (C_{i2} + \Delta C_{i2})x(t - d_i(t)) + (D_i + \Delta D_i)w(t) \]

\[ z(t) = (L_i + \Delta L_i)x(t), \quad i = 1, 2, \cdots, r, \]
Remark 1 When $E$ is a unit matrix, system (1) in this paper was studied in [9]. This paper spread the T-S fuzzy model into a more generalized condition.

Let $\xi(t) = [\xi_1(t), \xi_2(t), \cdots, \xi_p(t)]^T$, by using a center-average defuzzifier, product fuzzy inference and singleton fuzzifier, we have the global model of the system as the following form:

$$
E\dot{x}(t) = \sum_{i=1}^{r} h_i(\xi(t)) [(A_{1i} + \Delta A_{1i})x(t) + (A_{2i} + \Delta A_{2i})x(t - d_i(t)) + (B_i + \Delta B_i)w(t)]
$$

$$
y(t) = \sum_{i=1}^{r} h_i(\xi(t)) [(C_{1i} + \Delta C_{1i})x(t) + (C_{2i} + \Delta C_{2i})x(t - d_i(t)) + (D_i + \Delta D_i)w(t)]
$$

$$
z(t) = \sum_{i=1}^{r} h_i(\xi(t)) [(L_{1i} + \Delta L_{1i})x(t)],$$

\[ (6) \]

Remark 2 When $E$ is a unit matrix, and $C_{fi} = 0$, the fuzzy filter (5) is the same as the filter in [9]. This paper improve the function of the filter.

We rewrite (1) and (5) in the following compact form:

$$
E\ddot{x}(t) = \sum_{i=1}^{r} \sum_{j=1}^{r} h_i(\xi(t)) h_j(\xi(t)) [A_{ij}\dot{x}(t) + A_{di,j}\dot{x}(t - d_i(t)) + B_{ij}w(t)],
$$

$$
e(t) = \sum_{i=1}^{r} \sum_{j=1}^{r} h_i(\xi(t)) h_j(\xi(t)) [L_{ij}\dot{x}(t)],$$

\[ (6) \]

where

$$
e(t) = z(t) - \dot{z}(t), \dot{x}(t) = [x^T(t), \dot{x}^T(t)]^T, 
$$

$$
A_{ij} = \begin{bmatrix} A_{1i} + \Delta A_{1i} & 0 \\ B_{fj}C_{1i} + B_{fj}\Delta C_{1i} & A_{fj} \end{bmatrix}, A_{di,j} = \begin{bmatrix} A_{2i} + \Delta A_{2i} & 0 \\ B_{fj}(C_{2i} + \Delta C_{2i}) & C_{fj} \end{bmatrix},
$$

$$
B_{ij} = \begin{bmatrix} B_i + \Delta B_i \\ B_{fj}(D_i + \Delta D_i) \end{bmatrix}, \tilde{E} = \begin{bmatrix} E & 0 \\ 0 & E \end{bmatrix}, L_{ij} = \begin{bmatrix} L_i + \Delta L_i & -L_{fj} \end{bmatrix}.
$$

System (6) is simply denoted by the following form:

$$
\ddot{E}x(t) = \bar{A}\ddot{x}(t) + \bar{A}_d\dot{x}(t - d_i(t)) + \bar{B}w(t),
$$

\[ (7) \]
where
\[
\tilde{A} = \sum_{i=1}^{r} \sum_{j=1}^{r} h_i(\xi(t))h_j(\xi(t))A_{ij}, \quad \tilde{A}_d = \sum_{i=1}^{r} \sum_{j=1}^{r} h_i(\xi(t))h_j(\xi(t))A_{dij},
\]
\[
\tilde{B} = \sum_{i=1}^{r} \sum_{j=1}^{r} h_i(\xi(t))h_j(\xi(t))B_{ij}, \quad \tilde{L} = \sum_{i=1}^{r} \sum_{j=1}^{r} h_i(\xi(t))h_j(\xi(t))L_{ij}.
\]

Based on the above discussion, the problem to be addressed in this paper is stated as follows.

Given a finite scalar \( \gamma > 0 \), design a filter of form (5) such that the estimation error defined in (6) satisfies the \( H_\infty \) performance, i.e., (i) the augmented system (6) with \( w(t) = 0 \) is asymptotically stable; and (ii) the \( H_\infty \) performance \( \|e(t)\|_2 \leq \gamma \|w(t)\|_2 \) is satisfied for zero initial conditions and all nonzero \( w(t) \in L_2[0, \infty) \) with a prescribed \( \gamma > 0 \).

The following systems are regular consistent and with no pulse.

3 Main Results

**Lemma 1** [13] Given a set of suited dimension real matrices \( Q, H, E, Q \) is a symmetric matrix such that \( Q + HFE + E^T F^T H^T < 0 \) for all \( F \) satisfies \( F^T F \leq I \) if and only if there exists \( \varepsilon > 0 \) such that
\[
Q + \varepsilon HH^T + \varepsilon^{-1} E^T E < 0.
\]

**Theorem 1** Given a finite scalar \( \gamma > 0 \), the robust \( H_\infty \) filtering problem is solved for system (7) if there exists symmetric positive definite matrices \( W, U, \) and invertible matrix \( P \), such that
\[
\tilde{E}^T P = P^T \tilde{E} \geq 0,
\]
\[
\Sigma = \begin{bmatrix}
\Sigma_{11} & P^T \tilde{A}_d & 0 & P^T \tilde{B} \\
* & -(1 - \mu) \tilde{E}^T W H \tilde{E} & 0 & 0 \\
* & * & -d_2^{-1} U & 0 \\
* & * & * & -\gamma^2 I
\end{bmatrix} < 0,
\]
where
\[
\Sigma_{11} = \tilde{A}^T P + P^T \tilde{A} + \tilde{E}^T H^T (W + d_2 U) H \tilde{E} + \tilde{L}^T \tilde{L}.
\]

**Proof:** Choose a Lyapunov function for system (7) as
\[
V(\tilde{x}(t)) = \tilde{x}^T(t) \tilde{E}^T P \tilde{x}(t) + \int_{t-d(t)}^{t} \tilde{x}^T(s) \tilde{E}^T W H \tilde{E} \tilde{x}(s) ds + \int_{t-d(t)}^{t} \int_{t-d(t)}^{t} \tilde{x}^T(s) \tilde{E}^T U H \tilde{E} \tilde{x}(s) ds d\theta
\]
where \( W, U \) are symmetric positive definite matrices to be determined, and \( P \) is an invertible matrix to be determined, \( H = \begin{bmatrix} I & I \end{bmatrix} \). When \( w(t) = 0 \)
\[
\dot{V}(\tilde{x}(t)) \leq \begin{bmatrix}
\tilde{x}(t) \\
\tilde{x}(t - d(t)) \\
\int_{t-d(t)}^{t} H \tilde{E} \tilde{x}(s) ds
\end{bmatrix}^T \begin{bmatrix}
\Sigma_{11} & P^T \tilde{A}_d & 0 \\
* & \Sigma_{22} & 0 \\
* & * & -d_2^{-1} U
\end{bmatrix} \begin{bmatrix}
\tilde{x}(t) \\
\tilde{x}(t - d(t)) \\
\int_{t-d(t)}^{t} H \tilde{E} \tilde{x}(s) ds
\end{bmatrix},
\]
where
\[
\Sigma_{11} = \tilde{A}^T P + P^T \tilde{A} + \tilde{E}^T H^T (W + d_2 U) H \tilde{E} + \tilde{L}^T \tilde{L}.
\]
where $\Sigma'_{11} = \tilde{A}^T P + P^T \tilde{A} + \tilde{E}^T H^T (W + d_2 U) H \tilde{E}, \Sigma'_{22} = -(1 - \mu)\tilde{E}^T H^T W H \tilde{E}$. By (8b) we have $\dot{V}(\tilde{x}(t)) < 0$ in (10), the augmented system (7) is asymptotically stable. Furthermore, we consider the $H_\infty$ performance. Consider the Lyapunov function of the augmented system (7), we have

$$\dot{V}(\tilde{x}(t)) \leq \begin{bmatrix} \dot{x}(t) \\ \dot{x}(t - d_i(t)) \\ \int_{t-d_2}^t H \tilde{E} \tilde{x}(s) ds \\ w(t) \end{bmatrix}^T \begin{bmatrix} \Sigma'_{11} & P^T \tilde{A}_d & 0 & P^T \tilde{B} \\ * & \Sigma'_{22} & 0 & 0 \\ * & * & -d_2^{-1}U & 0 \\ * & * & * & 0 \end{bmatrix} \begin{bmatrix} \dot{x}(t) \\ \dot{x}(t - d_i(t)) \\ \int_{t-d_2}^t H \tilde{E} \tilde{x}(s) ds \\ w(t) \end{bmatrix}.$$

For zero initial condition

$$J = \int_0^T [e^T(t)e(t) - \gamma^2 w^T(t)w(t)]dt $$

$$= \int_0^T [e^T(t)e(t) - \gamma^2 w^T(t)w(t) + \dot{V}(\tilde{x}(t))]dt - V(\tilde{x}(t))$$

$$\leq \int_0^T [e^T(t)e(t) - \gamma^2 w^T(t)w(t) + \dot{V}(\tilde{x}(t))]dt$$

$$\leq \int_0^T x^T_\alpha \Sigma x_\alpha dt,$$

where $x_\alpha = \begin{bmatrix} \tilde{x}^T(t) & \tilde{x}^T(t - d_i(t)) & \left( \int_{t-d_2}^t H \tilde{E} \tilde{x}(s) ds \right)^T \end{bmatrix}^T w^T(t)$. When $\Sigma < 0$, for all $T > 0$ and $0 \neq w(t) \in L_2[0, \infty)$, we have $J < 0$. This completes the proof.

Based on the sufficient conditions above, the design problem of robust $H_\infty$ filter can be transformed into a problem of linear matrix inequality.

**Theorem 2** Given a finite scalar $\gamma > 0$, the robust $H_\infty$ filtering problem is solved for system (6) if there exists constant $\varepsilon_{11} > 0, \varepsilon_{2i} > 0$, symmetric positive definite matrix $W, U$ and invertible matrix $P = \text{diag}(P_1, P_2)$, such that

$$E^T P_1 = P_1^T E \geq 0, E^T P_2 = P_2^T E \geq 0,$$

$$\begin{bmatrix} \Gamma_1 & \Gamma_2 \\ * & -\varepsilon_{11}I \end{bmatrix} < 0, \begin{bmatrix} \bar{\Gamma}_1 & \bar{\Gamma}_2 \\ * & -\varepsilon_{2i}I \end{bmatrix} < 0,$$

where

$$\Gamma_1 = \begin{bmatrix} \eta_{11} & \eta_{12} & \eta_{13} & 0 & 0 & \eta_{16} & L_i^T \\ * & \eta_{22} & M_{2i} C_{2i} & M_{3i} & 0 & M_{2i} D_i & -L_i^T \\ * & * & \eta_{33} & -(1 - \mu)E^T WE & 0 & \varepsilon_{11} E_{2i}^T E_{3i} & 0 \\ * & * & * & -(1 - \mu)E^T WE & 0 & 0 & 0 \\ * & * & * & * & -d_2^{-1}U & 0 & 0 \\ * & * & * & * & * & \eta_{66} & 0 \\ * & * & * & * & * & * & -I \end{bmatrix},$$
The parameters of the robust $H\infty$ filter are

$$A_{fi} = (P_2^{-1})^T M_{1i}, \quad B_{fi} = (P_2^{-1})^T M_{2i}, \quad C_{fi} = (P_2^{-1})^T M_{3i}, \quad L_{fi},$$

**Remark 3** Theorem 2 provides a solution to the robust $H\infty$ filter. For the delay state vector,
in [9] the time delay $d$ is a constant but unknown. While in this paper, the time delay $d_i(t)$ is time-varying and bounded. The proposed approach is more applicable.

4 Numerical Value Examples

Example Consider the system (1) described by

$$
E = \begin{bmatrix} 0.1 & 0.2 \\ 0.3 & 0.6 \end{bmatrix},
A_{11} = \begin{bmatrix} 0.1 & 0 \\ 0.1 & 0 \end{bmatrix},
A_{21} = \begin{bmatrix} 0.2 & 0.1 \\ 0.1 & 0.2 \end{bmatrix},
B_1 = \begin{bmatrix} 0.3 & 0.2 \\ 0.4 & 0.1 \end{bmatrix},
C_{11} = \begin{bmatrix} 0.1 & 0.3 \\ 0.1 & 0.2 \end{bmatrix},
C_{21} = \begin{bmatrix} 0.2 & 0.4 \\ 0.1 & 0.2 \end{bmatrix},
D_1 = \begin{bmatrix} 0.1 & 0.2 \\ 0.1 & 0.1 \end{bmatrix},
H_{11} = \begin{bmatrix} 0.1 \\ 0.2 \end{bmatrix},
H_{21} = \begin{bmatrix} -0.2 \\ 0.2 \end{bmatrix},
H_{31} = \begin{bmatrix} -0.2 \\ 0.1 \end{bmatrix},
L_1 = \begin{bmatrix} 0.2 & 0.1 \\ 0.1 & 0.2 \end{bmatrix},
E_{11} = \begin{bmatrix} -0.2 & 0.1 \end{bmatrix},
E_{21} = \begin{bmatrix} 0.2 & -0.2 \end{bmatrix},
E_{31} = \begin{bmatrix} -0.2 & 0.1 \end{bmatrix},
E_{41} = \begin{bmatrix} -0.2 & 0.2 \end{bmatrix},
A_{12} = \begin{bmatrix} 0 & 0.1 \\ 0.1 & 0.2 \end{bmatrix},
A_{22} = \begin{bmatrix} 0.2 & 0.1 \\ 0.1 & 0.3 \end{bmatrix},
B_2 = \begin{bmatrix} 0.7 & 0.1 \\ 0.2 & 0.1 \end{bmatrix},
C_{12} = \begin{bmatrix} 0.1 & 0.4 \\ 0.3 & 0.2 \end{bmatrix},
C_{22} = \begin{bmatrix} 0.3 & 0.4 \\ 0.1 & 0.2 \end{bmatrix},
D_2 = \begin{bmatrix} 0.2 & 0.3 \\ 0.1 & 0.4 \end{bmatrix},
H_{12} = \begin{bmatrix} 0.2 \\ 0.1 \end{bmatrix},
H_{22} = \begin{bmatrix} 0.2 \\ 0.1 \end{bmatrix},
H_{32} = \begin{bmatrix} -0.2 \\ 0.3 \end{bmatrix},
L_2 = \begin{bmatrix} 0.1 & 0.2 \\ 0.3 & 0.1 \end{bmatrix},
E_{12} = \begin{bmatrix} 0.2 & 0.1 \end{bmatrix},
E_{22} = \begin{bmatrix} -0.1 & -0.2 \end{bmatrix},
E_{32} = \begin{bmatrix} -0.3 & 0.2 \end{bmatrix},
E_{42} = \begin{bmatrix} -0.2 & 0.3 \end{bmatrix}.
$$

The normalized membership functions of the first subsystem are

$$
h_1(x(t)) = \frac{1+\cos(x(t))}{2}, \ h_2(x(t)) = \frac{1-\cos(x(t))}{2}.
$$

Given $\mu = 1$, $d_2 = 2$ and $\gamma = 0.75$. We can obtain the filter parameters as follows:

$$
A_{f1} = \begin{bmatrix} 2.8204 & -1.6035 \\ -1.6120 & 0.2120 \end{bmatrix},
B_{f1} = \begin{bmatrix} 0.0007 & 0.0020 \\ 0.0002 & -0.0001 \end{bmatrix},
C_{f1} = 1.0e-003 \begin{bmatrix} 0.3086 & 0.6172 \\ -0.0972 & -0.1943 \end{bmatrix},
L_{f1} = \begin{bmatrix} -8.5008 & -0.4930 \\ -14.6899 & -0.7322 \end{bmatrix},
A_{f2} = \begin{bmatrix} 2.8056 & -1.6377 \\ -1.6560 & 0.1245 \end{bmatrix},
B_{f2} = \begin{bmatrix} -0.0001 & -0.0014 \\ 0.0002 & 0.0013 \end{bmatrix},
C_{f2} = 1.0e-003 \begin{bmatrix} -0.2044 & -0.4088 \\ 0.0292 & 0.0585 \end{bmatrix},
L_{f2} = \begin{bmatrix} 2.0533 & 3.3366 \\ 3.5127 & 5.8994 \end{bmatrix}.
$$
5 Conclusion

This paper based on linear matrix inequality, studied a design problem of robust delay-dependent $H_{\infty}$ filter for a class of T-S fuzzy descriptor time-delay systems with parameter uncertainties. At first, the design of robust delay-dependent $H_{\infty}$ filter for a class of T-S fuzzy descriptor time-delay systems with parameter uncertainties is simplified to a general delay systems with parameter determined. And the problem of robust $H_{\infty}$ filter for parameter determines can be solved by linear matrix inequalities. The results can be easily extended to systems with multiple delays, and the filtering is asymptotically stable for all the augmented system with uncertainty. The numerical example illustrates the effectiveness of the method proposed.

References