Representation and Reasoning for Topological Relations between a Region with Broad Boundaries and a Simple Region on the Basis of RCC-8

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Abstract

In this article, on the basis of RCC-8, the 9-intersection matrix is extended to 27-intersection matrix, to represent the topological relations between a region with broad boundaries and a simple region. We get 23 topological relations between a region with broad boundaries and a simple region, and give its topological relations diagram. For further study 27-intersection model, we give the conceptual neighborhood diagram and topology reasoning composite table and verify that the 23 kinds of topological relations are implemented.

Keywords: Artificial Intelligence; Topological Relations; RCC-8; Simple Region; Regions with Broad Boundaries; 27-Intersection Matrix

1 Introduction

The broad boundary refers to a boundary of the region that has a certain width and area, and is no longer a pure geometric sense line. When researching the wide boundary region, the traditional 9-intersection model cannot handle the topological relations of regions with broad boundaries, Clementini et al. propose the extended 9-intersection model [1], Cohn et al. propose “Egg yolk” theory [2]. The above model can only represent part of the topological relations.

In order to establish a research model that is suitable to represent the topological relations between a region with broad boundaries and a simple region, based on the RCC-8, we extend the 9-intersection matrix and then give a 27-intersection model.

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2 Representation for Topological Relations between a Region with Broad Boundaries and a Simple Region

2.1 RCC theory

In 1992, Randell et al. gave the RCC theory [3, 4]. Afterwards RCC theory has been further studied and obtained improvement such as RCC-8 relations and RCC-5 relations (As shown in Figure 1).

![RCC Relations Diagram]

**Fig. 1:** The topological relations between two simple regions

2.2 9-Intersection model

In 1991, Egenhofer et al. constructed 4-intersection matrix model, here \(A^0\) means the interior of \(A\) and \(\partial A\) means the boundary of \(A\).

\[
\begin{pmatrix}
A^0 \cap B^0 & A^0 \cap \partial B \\
\partial A \cap B^0 & \partial A \cap \partial B
\end{pmatrix}
\]

Based on the 4-intersection model, the complement \(A^-\) of region \(A\) is regarded as the exterior of region \(A\), we extend the 4-intersection matrix to 9-intersection matrix, as follows:

\[
\begin{pmatrix}
A^0 \cap B^0 & A^0 \cap \partial B & A^0 \cap B^- \\
\partial A \cap B^0 & \partial A \cap \partial B & \partial A \cap B^- \\
A^- \cap B^0 & A^- \cap \partial B & A^- \cap B^-
\end{pmatrix}
\]

When considering objectives in the real world, we obtain 8 topological relations corresponding to above RCC-8 relations. (As shown in Figure 2).

![9-Intersection Model and 8 Topological Relations]

**Fig. 2:** 9-intersection model and 8 topological relations
2.3 Regions with broad boundaries

**Definition 1** Region $A$ with broad boundaries includes two precise regions $A_1$ and $A_2$, satisfies the condition $A_1 \subset A_2$.

Where $A_1$ is the internal region, $A_2$ is the external region. If $A_1$ is the proper subset of $A_2$, region $A$ has broad boundaries; otherwise $A$ is a precise region [5]. As shown in Figure 3:

![Fig. 3: A region with broad boundaries](image)

**Definition 2** A region with simple broad boundaries and a simple region, as shown in Figure 4, $C$ is the internal region, $B$ is the external region, $A$ is the simple region.

![Fig. 4: Simple region with broad boundaries and a simple region](image)

2.4 27-Intersection model

Based on 9-intersection model, space region is divided into nine sections, as shown in Figure 5. Similarly, three regions of $A$, $B$, $C$ can divide the space into 27 parts, in other word, we can get

27-intersection matrix, as follows:

$$
\begin{array}{ccc}
    A^0 \cap B^0 \cap C^0 & A^0 \cap B^0 \cap \partial C & A^0 \cap B^0 \cap C^- \\
    A^0 \cap \partial B \cap C^0 & A^0 \cap \partial B \cap \partial C & A^0 \cap \partial B \cap C^- \\
    A^0 \cap \partial B^0 \cap C^0 & A^0 \cap B^0 \cap \partial C & A^0 \cap B^0 \cap C^- \\
    \partial A \cap B^0 \cap C^0 & \partial A \cap B^0 \cap \partial C & \partial A \cap B^0 \cap C^- \\
    \partial A \cap \partial B \cap C^0 & \partial A \cap \partial B \cap \partial C & \partial A \cap \partial B \cap C^- \\
    \partial A \cap \partial B^0 \cap C^0 & \partial A \cap \partial B^0 \cap \partial C & \partial A \cap \partial B^0 \cap C^- \\
    A^- \cap B^0 \cap C^0 & A^- \cap B^0 \cap \partial C & A^- \cap B^0 \cap C^- \\
    A^- \cap \partial B \cap C^0 & A^- \cap \partial B \cap \partial C & A^- \cap \partial B \cap C^- \\
    A^- \cap \partial B^0 \cap C^0 & A^- \cap \partial B^0 \cap \partial C & A^- \cap \partial B^0 \cap C^- \\
\end{array}
$$

![Fig. 5: 9 part of the space is divided](image)
we can get $\varepsilon$.

**Definition 4** C is a one-to-one correspondence.

For region $D$ in Definition 3, the 2-set-intersection matrix can be regarded as $M$.

**Definition 3** For region $A$, define $\varepsilon(A) = \begin{cases} 1, & A \text{ is not empty,} \\ 0, & A \text{ is empty.} \end{cases}$ Then $M$ is regarded as $M_\varepsilon$.

$$M = \begin{pmatrix} M_{000} & M_{001} & M_{002} \\ M_{010} & M_{011} & M_{012} \\ M_{020} & M_{021} & M_{022} \\ M_{100} & M_{101} & M_{102} \\ M_{110} & M_{111} & M_{112} \\ M_{120} & M_{121} & M_{122} \\ M_{200} & M_{201} & M_{202} \\ M_{210} & M_{211} & M_{212} \\ M_{220} & M_{221} & M_{222} \end{pmatrix}, \quad M_\varepsilon = \begin{pmatrix} \varepsilon(M_{000}) & \varepsilon(M_{001}) & \varepsilon(M_{002}) \\ \varepsilon(M_{010}) & \varepsilon(M_{011}) & \varepsilon(M_{012}) \\ \varepsilon(M_{020}) & \varepsilon(M_{021}) & \varepsilon(M_{022}) \\ \varepsilon(M_{100}) & \varepsilon(M_{101}) & \varepsilon(M_{102}) \\ \varepsilon(M_{110}) & \varepsilon(M_{111}) & \varepsilon(M_{112}) \\ \varepsilon(M_{120}) & \varepsilon(M_{121}) & \varepsilon(M_{122}) \\ \varepsilon(M_{200}) & \varepsilon(M_{201}) & \varepsilon(M_{202}) \\ \varepsilon(M_{210}) & \varepsilon(M_{211}) & \varepsilon(M_{212}) \\ \varepsilon(M_{220}) & \varepsilon(M_{221}) & \varepsilon(M_{222}) \end{pmatrix}.$$ 

So for any a region with broad boundaries and a simple region, we can represent the topological relations by a 9×3 0-1 matrix, and these 0-1 matrix and topological relations between the $A$, $B$, $C$ is a one-to-one correspondence.

### 2.5 Properties of 2-set-intersection model

**Definition 4** We define an operation “∨” on the set $\{0, 1\}$, Table 1 below:

<table>
<thead>
<tr>
<th>$\lor$</th>
<th>0</th>
<th>1</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0</td>
<td>1</td>
</tr>
<tr>
<td>1</td>
<td>1</td>
<td>1</td>
</tr>
</tbody>
</table>

For any $m \times n$ 0-1 matrices $A = (a_{ij})_{m \times n}$ and $B = (b_{ij})_{m \times n}$, define $A \lor B = (a_{ij} \lor b_{ij})_{m \times n}$. So we can get $\varepsilon(A \lor B) = \varepsilon(A) \lor \varepsilon(B)$. Order RCC-8 to $\Omega$, among

$$\Omega = \left( \begin{pmatrix} 0 & 0 & 1 \\ 0 & 0 & 1 \\ 0 & 0 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{pmatrix}, \begin{pmatrix} 1 & 1 & 0 \\ 1 & 1 & 0 \\ 1 & 1 & 0 \\ 1 & 1 & 0 \\ 1 & 1 & 0 \\ 1 & 1 & 0 \\ 1 & 1 & 0 \\ 1 & 1 & 0 \end{pmatrix}, \begin{pmatrix} 0 & 0 & 1 \\ 0 & 0 & 1 \\ 0 & 0 & 1 \\ 0 & 0 & 1 \\ 0 & 0 & 1 \\ 0 & 0 & 1 \\ 0 & 0 & 1 \\ 0 & 0 & 1 \end{pmatrix} \right),$$

For any two simple regions $X$ and $Y$, the corresponding matrix of their topological relations must belong to the set $\Omega$. So we can get theorem 1:

**Theorem 1** A region with broad boundaries and a simple region, each region $A$, $B$, $C$, $D$ is described as showed in Figure 4, then there must be

$$\begin{pmatrix} \frac{3}{2} \varepsilon(M_{000}) & \frac{3}{2} \varepsilon(M_{010}) & \frac{3}{2} \varepsilon(M_{020}) \\ \frac{3}{2} \varepsilon(M_{001}) & \frac{3}{2} \varepsilon(M_{011}) & \frac{3}{2} \varepsilon(M_{021}) \\ \frac{3}{2} \varepsilon(M_{002}) & \frac{3}{2} \varepsilon(M_{012}) & \frac{3}{2} \varepsilon(M_{022}) \end{pmatrix} = \begin{pmatrix} A^0 \cap B^0 & A^0 \cap \partial B & A^0 \cap B^- \\ \partial A \cap B^0 & \partial A \cap \partial B & \partial A \cap B^- \\ \partial A \cap B^- & A^- \cap \partial B & A^- \cap B^- \end{pmatrix} \in \Omega.$$
Here \( \forall x \in (M_{000}) = \varepsilon (M_{000}) \lor \varepsilon (M_{001}) \lor \varepsilon (M_{002}) \). Similarly we can get the formula

\[
\left( \begin{array}{c}
\frac{2}{x} \varepsilon (M_{000}) \\
\frac{2}{x} \varepsilon (M_{100}) \\
\frac{2}{x} \varepsilon (M_{200})
\end{array} \right) \left( \begin{array}{c}
\frac{2}{x} \varepsilon (M_{001}) \\
\frac{2}{x} \varepsilon (M_{111}) \\
\frac{2}{x} \varepsilon (M_{211})
\end{array} \right) \left( \begin{array}{c}
\frac{2}{x} \varepsilon (M_{002}) \\
\frac{2}{x} \varepsilon (M_{112}) \\
\frac{2}{x} \varepsilon (M_{212})
\end{array} \right) = (A \cap C)^0 \lor A \cap \partial C \lor A^0 \cap C^- \lor \partial A \cap C^0 \lor \partial A \cap C^- \lor A^- \cap C^0 \lor A^- \cap C^-)
\in \Omega,
\]

Since the intersection of two sets is either empty or nonempty, we can obtain the following theorem.

**Theorem 2** The topological relations of three regions given by 27-intersection model are exclusive and complete.

### 2.6 Constraints of topological relations

**Restricted condition 1:** A 0-1 matrix corresponding to an achievable three topological relations must satisfy theorem 1. In other word, must satisfy the three formulas.

**Restricted condition 2:** For simple bounded regions, \( A^- \cap B^- \cap C^- \) must be nonempty, i.e. \( M_{222} = 1 \).

**Restricted condition 3:** For a region with broad boundaries and a simple region, by the definition of the regions with broad boundaries, we know that the region \( B \) contains the region \( C \) in Figure 4, so we can get the following expression:

\[
\left( \begin{array}{c}
\frac{2}{x} \varepsilon (M_{000}) \\
\frac{2}{x} \varepsilon (M_{100}) \\
\frac{2}{x} \varepsilon (M_{200})
\end{array} \right) \left( \begin{array}{c}
\frac{2}{x} \varepsilon (M_{001}) \\
\frac{2}{x} \varepsilon (M_{111}) \\
\frac{2}{x} \varepsilon (M_{211})
\end{array} \right) \left( \begin{array}{c}
\frac{2}{x} \varepsilon (M_{002}) \\
\frac{2}{x} \varepsilon (M_{112}) \\
\frac{2}{x} \varepsilon (M_{212})
\end{array} \right) = (B \cap C^0 \lor B \cap C \lor B \cap C^- \lor \partial B \cap C \lor \partial B \cap C^- \lor B^- \cap C \lor B^- \cap C^-)
\in \Omega.
\]

### 2.7 Topology diagram

According to the above three constraints, we can give special program, 23 0-1 matrices were gotten through the program, and verified these can uniquely correspond to the topological relations between a region with broad boundaries and a simple region. According to the 23 kinds of topological relations matrix, we specifically draw this 23 topology diagram, as shown in Figure 6.

### 3 The Reasoning of Topological Relations

Representing the topological relations between region \( A \) and region \( B \) by using \( R(A,B) \). Reasoning of topological relations is that when we know the topological relations of one or more regions, then reasoning the topological relations of unknown regions. By researching the topological relations between a region with broad boundaries and a simple region, we can get 23 kinds
of topological relations. If known \( R(A, B) \), we can reasoning \( R(A, C) \). For example, if known \( R(A, B) = \text{meet} \), \( R(A, C) = \text{disjoint} \) can be obtained; If known \( R(A, B) = \text{overlap} \), \( R(A, C) = \text{disjoint, meet, overlap, covers, contains} \) can be obtained. Therefore, we can obtain the topological relations composition table as shown in Table 2.

<table>
<thead>
<tr>
<th>( R(A, B) )</th>
<th>( R(A, C) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>disjoint</td>
<td>disjoint</td>
</tr>
<tr>
<td>meet</td>
<td>disjoint</td>
</tr>
<tr>
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<td>contains</td>
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<td>covers</td>
<td>contains</td>
</tr>
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<td>contains</td>
<td>contains</td>
</tr>
</tbody>
</table>

### 4 Conceptual Neighborhood Graph

In order to further complete the reasoning of topological relations, we give the conceptual neighborhood graph of 23 kinds of topological relations, as shown in Figure 7, using the straight line connected two rectangular represents two topological relations by step into each other. Corresponding to the box is serial number and topological relations of region \( A \) and \( B, C \), the conceptual neighborhood graph laid the foundation for further study 27-intersection model.

### 5 Comparison with Related Research Work

According to the extended 9-intersection model can only get 14 kinds of topological relations. By the extended 9-intersection model, a region with broad boundaries is divided into three parts, as shown in Figure 8.
Under this division is to study the topological relations between a region with broad boundaries and a simple region, can be found that some cases cannot be distinguished, for example, the four situations in Figure 9 cannot be distinguished by the extended 9-intersection model, and is considered as one situation, other similar situation that cannot be distinguished has a lot.

According to this model, we can get 23 kinds of topological relations, than before, more than 9 kinds of situations, and distinct the situation cannot be distinguished, indicate that this model is better.

6 Conclusions

Based on RCC-8, 9-intersection matrix is defined, thus giving 27-intersection model, representing the topological relations between a region with broad boundaries and a simple region. Give the topological relations diagram and the conceptual neighborhood diagrams of 23 topological relations between a region with broad boundaries and a simple region. By the conceptual neighborhood graph, laid the foundation for further study 27-intersection model

References


