Large Disturbance Attenuation Sampled-data Controller Design for Nonlinear STATCOM System

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Abstract

A novel large disturbance attenuation sampled-data control scheme is investigated for the strongly nonlinear static synchronous compensator (STATCOM) system using backstepping technique. Focus is centered on the effect of unknown large disturbances that occurs on the system. The Minimax method is used to compute the maximum degree of disturbance. This method effectively reduces the conservativeness brought by the existing disturbance treatment, and ensure the insensitivity to large disturbances. Furthermore, considering the effect of mechanical power disturbance and short-circuit ground fault, the designed scheme was validated by simulation. Compared with a continuous controller designed from the same method, the results showed that the proposed controller has better behaviors on enhancing the transient stability of power systems.

Keywords: Disturbance Attenuation; Sampled-data Stabilization; Minimax; Backstepping; STATCOM

1 Introduction

With the increasing demand for electrical power, a static synchronous compensator (STATCOM) will be the power electronic solution with the best performance at low voltages [1]. STATCOM can affect rapid control of reactive flow in the transmission line by controlling the generated AC voltage [2]. Recently, advanced control theory and design method, aiming at application research on transient stability of systems, have been introduced to the STATCOM control devices, which effectively improves the recovery of system stability [3-5]. Backstepping is a Lyapunov-based scheme for nonlinear feedback system [6-8], which can be used to the STATCOM system.

Instability of a power system is often triggered by large disturbances, so adopt more effective control strategy for perturbation attenuation is meaningful. The Minimax method is an effective
approach to address large disturbance attenuation and uncertain problems [9, 10]. The in-depth study on the degree of the disturbance influence on power system has been conducted [11, 12].

The prevalence of digital control leads to a limitation of the utilization of traditional continuous-control methods. This process promotes investigation of the so-called sampled-data systems consisting of a continuous-time plant or process controlled by a discrete-time controller. The plant and the controller are interconnected via the analog-to-digital (A-D) and digital-to-analog (D-A) converters[14-15]. Accurate stabilizations of sampled-data nonlinear systems are proposed via discrete-time approximations [16]. Several Backstepping designs are presented based on the Euler approximate discrete-time model of a continuous-time plant in strict feedback form [17].

This paper proposes a sampled-data stabilization controller for STATCOM system based on the nonlinear approximate discrete-time model of the original system, which is more suitable for present digital-control industry. The advantage of the approach lies in the using of the Minimax method to deal with the disturbance in the classical Backstepping algorithm. The proposed method estimates the worst case of the disturbances, which avoid estimating the upper bound or inequality scaling the disturbance. The designed controller ensures quick convergence of the trajectory to a stable equilibrium point under any fault and large disturbances. Simulation demonstrates that the sampled-data stabilization controller possesses superior performances with respect to continuous-time nonlinear controller.

1.1 Dynamic model and problem statements

The mathematical dynamic model of a single-machine infinite-bus system with STATCOM is expressed as follows.

\[
\dot{\delta} = \omega - \omega_0 \\
\dot{\omega} = \frac{\omega_0}{H} \left[ \frac{P_m}{\omega_0} (\omega - \omega_0) - \frac{E'_q V_s \sin \delta}{X_1 + X_2} \left[ 1 + \frac{X_1 X_2 I_q}{\sqrt{(X_2 E'_q)^2 + (X_1 V_s)^2 + 2X_1 X_2 E'_q V_s \cos \delta}} \right] \right] \\
\dot{I}_q = \frac{1}{T_q} (-I_q + I_{q0} + u)
\]

where \(\delta\) is the rotor angle of generator; \(\omega\) is the rotor speed; \(P_m\) is mechanical power of the prime motor; \(D\) is the damping coefficient; \(E'_q\) is the transient EMF of the \(q\)-axis; \(H\) is the inertia; \(V_s\) is the infinite bus voltage; \(X_1 = X'_d + X_T + X_L\) and \(X_2 = X_L\) are the reactance of transmission lines, where \(X'_d\) is the \(d\)-axis transient reactance; \(X_T\) is the transformer reactance; \(X_L\) is the reactance of transmission line; \(I_q\) is the output reactive current of STATCOM and \(I_{q0}\) is its steady value; \(\omega_0\) is the initial value of \(\omega\); \(X'_d\) is the \(d\)-axis transient reactance; \(X_T\) is the transformer reactance; \(X_L\) is the reactance of transmission line; \(I_q\) is the output reactive current of STATCOM and \(I_{q0}\) is its steady value; \(\delta_0, \omega_0, I_{q0}\) are the initial values of the corresponding variables; suppose that \(\varepsilon_1\) and \(\varepsilon_2\) are the unknown disturbances. Thus, Eq. (1)
can be rewritten as follows:
\[
\begin{align*}
\dot{x}_1 &= x_2 \\
\dot{x}_2 &= k_1 x_2 + k_2 P_m - k_3 \sin (x_1 + \delta_0) [1 + f(x_1)(x_3 + I_0)] + \varepsilon_1 \\
\dot{x}_3 &= \frac{1}{T_q} (-x_3 + u) + \varepsilon_2 \\
z &= \begin{bmatrix} q_1 x_1 \\ q_2 x_2 \end{bmatrix}
\end{align*}
\]
where \( f(x_1) = \frac{X_1 X_2}{\sqrt{(X_2 E_o)^2 + (X_1 V_s)^2 + 2 X_1 X_2 E_o V_s \cos (x_1 + \delta_0)}} \); \( k_1 = -\frac{D}{H} \), \( k_2 = \frac{\omega_0}{H} \), \( k_3 = \frac{\omega_0 E_o V_s}{H(X_1 + X_2)} \) are constants. \( z = [q_1 x_1 \ q_2 x_2]^T \) is the regulation output, where \( q_1 \) and \( q_2 \) are the non-negative weight coefficients, that is, they can be considered as the weights between \( x_1 \) and \( x_2 \), and \( q_1^2 + q_2^2 \leq 1 \).

To design a sampled-data controller for a continuous-time nonlinear plant, we obtain the approximate discrete-time models based on Eqs. (2)-(3).

\[
\begin{align*}
x_1(k + 1) &= x_1(k) + T_s x_2(k) \\
x_2(k + 1) &= x_2(k) + T_s k_1 x_2(k) + T_s k_2 P_m - T_s k_3 \sin (x_1(k) + \delta_0) [1 + f(x_1(k))(x_3(k) + I_0)] + T_s \varepsilon_1(k) \\
x_3(k + 1) &= x_3(k) + \frac{T_s}{T_q} [-x_3(k) + u(k)] + T_s \varepsilon_2(k) \\
z(k) &= \begin{bmatrix} q_1 x_1(k) \\ q_2 x_2(k) \end{bmatrix}
\end{align*}
\]
where \( T_s > 0 \) is a sampling period that satisfies the approximation ratio.

## 2 Design of Sampled-data Stabilization Controller with Large-disturbance Attenuation

This article focuses on the effect of unknown large disturbances that occurs on the system. Generally, the disturbance is dealt with by artificially imposing an upper bound or inequality scaling. However, the traditional solutions and assumptions are sometimes unreasonable. Here we use the Minimax method to address the disturbance problem, which reduces effectively the conservativeness inherent in the traditional treatment. The Minimax and Backstepping controller design procedure are formulated as follows.

**Step 1:** For the first equation of Eq. (4), we define \( e_1(k) = x_1(k) \), thus

\[
e_1(k + 1) = x_1(k) + T_s x_2(k)
\]

We view \( x_2(k) \) as a control variable and define a virtual control law for Eq. (6), say \( x_2^*(k) \), and let \( e_2(k) \) be an error variable representing the difference between the actual and virtual controls

\[
e_2(k) = x_2(k) - x_2^*(k)
\]
Consider a control Lyapunov function as

\[ V_1(k) = \sigma e_1^2(k) \]  

where \( \sigma > 0 \). The difference of \( V_1(k) \) is found as:

\[ \Delta V_1(k) = V_1(k+1) - V_1(k) = \sigma [T_s e_2(k) + T_s x_2^2(k)][2x_1(k) + T_s e_2(k) + T_s x_2^2(k)] \]  

(9)

Let \( x_2^*(k) = -c_1 x_1(k), \) \( c_1 = 2/(T_s + 1) > 0. \) If \( e_2(k) = 0, \) then \( \Delta V_1(k) = -\sigma T_s x_2^2(k). \)

**Step 2:** We derive the error difference equation for \( e_2(k) = x_2(k) - x_2^*(k) \).

\[
e_2(k + 1) = x_2(k) + T_s k_1 x_2(k) + T_s k_2 P_m - T_s k_3 \sin (x_1(k) + \delta_0) \\
[1 + f(x_1(k))(x_3(k) + I_0)] + T_s \epsilon_1(k) + c_1 x_1(k)
\]  

(10)

in which \( x_3(k) \) is viewed as a virtual control input. Define a virtual control law \( x_3^*(k) \) and let \( e_3(k) \) be an error variable, \( e_3(k) = x_3(k) - x_3^*(k) \).

Augmenting Eq. (9), the new Lyapunov function can be expressed in the form

\[ V_2(k) = V_1(k) + e_2^2(k) \]  

(11)

We apply Minimax method to handle unknown large disturbance. We construct a test function prior to controller design, which is used to estimate the highest degree of critical disturbance that can be endured by the system. The energy function can be defined as

\[ H_1(k) = \Delta V_2(k) + \frac{1}{2}(\|z(k)\|^2 - \gamma^2 \|\epsilon_1(k)\|^2) \]  

(12)

The performance index can be expressed as follows:

\[ J_1(k) = \sum_{k=0}^{N} (\|z(k)\|^2 - \gamma^2 \|\epsilon_1(k)\|^2) \]

Substituting \( \Delta V_2(k) \) into Eq. (12) yields then one gets

\[ H_1(k) = \Delta V_1(k) + \{x_2(k) + T_s k_1 x_2(k) + T_s k_2 P_m - T_s k_3 \sin (x_1(k) + \delta_0)[1 + f(x_1(k))(I_0) + x_3(k)] + T_s \epsilon_1(k) + c_1 x_1(k))^2 - [x_2(k) + c_1 x_1(k)]^2 + \frac{q_1 x_1^2(k)}{2} + \frac{q_2 x_1^2(k)}{2} - \gamma^2 \frac{\epsilon_1^2(k)}{2} \]  

(13)

For each subsystem, if a disturbance exists and makes \( J_1(k) \) larger, then the degree of damage is greatest on the system performance. Assuming that the derivative of \( H_1(k) \) with respect to \( \epsilon_1(k) \) is zero, then

\[ \epsilon_1^*(k) = \frac{2T_s}{2T_s - \gamma^2} F_1(k) \]  

(14)

\[ F_1(k) = T_s k_3 \sin (\delta_0 + x_1(k))[1 + f(x_1(k))(x_3(k) + I_0)] - c_1 x_1(k) - (1 + c_1 T_s + T_s k_1) x_2(k) - T_s k_2 P_m \]

By calculating the second derivation of Eq. (13) and selecting the proper values of \( T_s \) and \( \gamma \), then \( \frac{\partial^2 H_1(k)}{\partial \epsilon_1^2(k)} = 2T_s^2 - \gamma^2 < 0. \) Thus, the maximum value of \( H_1(k) \) relative to \( \epsilon_1(k) \) exists, and

\[ \max H_1(k) = \max [\Delta V_2(k) + \frac{1}{2}(\|z(k)\|^2 - \gamma^2 \|\epsilon_1(k)\|^2)] \]  

(15)
Obtaining the summation from both sides of Eq. (15), we can have
\[
\max_{k=0}^{N} H_1(k) = \max_{k=0}^{N} \Delta V_2(k) + \frac{1}{2} \sum_{k=0}^{N} \left( \| z(k) \| - \gamma \| \varepsilon_1(k) \| \right)
\]

Let \( \tilde{H}_1(k) = \sum_{k=0}^{N} H_1(k) \), then \( \max \tilde{H}_1(k) = \max \left( \langle V_2(N) - V_2(0) \rangle + J_1(k)/2 \right) \). As \( J_1(k)/2 = \tilde{H}_1(k) - \Delta V_2(k) \), then \( \max (J_1(k)/2) = \max (\tilde{H}_1(k) - \Delta V_2(k)) \) \( \leq \max (\tilde{H}_1(k)) - \min (\Delta V_2(k)) \).

**Remark 1:** Disturbance \( \varepsilon_1(k) \) is assumed to reduce \( V_2(k) \) to zero, that is, \( \min (\Delta V_2) = 0 \). Thus, \( \varepsilon_1(k) \) is the worst disturbance for the system.

**Remark 2:** If \( \varepsilon_1(k) \) causes \( \tilde{H}_1(k) \) to reach the maximum value, then \( \varepsilon_1(k) \) also allows \( J_1(k) \) to attain the maximum value, that is, the system performance damage via \( \varepsilon_1(k) \) is the largest. Hence, the controller needs to be designed to undertake the substitution of the disturbances of such damage degree into the systems to ensure the stability of the closed-loop system.

Substituting Eq. (14) into Eq. (13), the following is obtained:
\[
H_1(k) = -c_1^2 - \frac{1}{2} q_1^2 - \frac{3}{2} x_1^2(k) - \left[ 1 - \frac{1}{2} q_2^2 - \sigma T_s^2 - \frac{(c_1 - \sigma T_s)^2}{3} \right] x_2^2(k) - [\sqrt{3} x_1(k) + \frac{c_1 - \sigma T_s}{\sqrt{3}} x_2(k)]^2 + (\frac{2 T_s^2}{2 T_s^2 - \gamma^2} - 1)^2 - \frac{1}{2} \gamma^2 (\frac{2 T_s^2}{2 T_s^2 - \gamma^2}) F_1^2(k)
\]

suppose that \( c_2 = (\frac{2 T_s^2}{2 T_s^2 - \gamma^2} - 1)^2 - \frac{1}{2} \gamma^2 (\frac{2 T_s^2}{2 T_s^2 - \gamma^2})^2 \), \( M_1 = c_2 - \frac{1}{2} q_2^2 - 3 \), \( M_2 = 1 - \frac{1}{2} q_2^2 - \sigma T_s^2 - \frac{(c_1 - \sigma T_s)^2}{3} \).

By selecting the proper positive numbers \( \gamma, T_s, \sigma, c_1, q_1, q_2 \), let \( c_2 > 0, M_1 > 0, M_2 > 0 \). We can obtain \( H_1(k) = -M_1 x_1^2(k) - M_2 x_2^2(k) - [\sqrt{3} x_1(k) + \frac{c_1 - \sigma T_s}{\sqrt{3}} x_2(k)]^2 + c_2 F_1^2(k) \).

Selecting an appropriate virtual consider \( x_3^*(k) \) to ensure \( H_1(k) \leq 0 \), that is let \( F_1(k) = 0 \), then:
\[
x_3^*(k) = \frac{n_1(k) F_2(k)}{1 + f(x_1(k))} - I_{q0}
\]

where \( n_1(k) = \frac{1}{T_{e_{k3}} \sin (\delta_0 + x_1(k))} \) (Because \( 0 < \delta < 2\pi, \sin (\delta_0 + x_1(k)) \neq 0) \); \( F_2(k) = c_1 x_1(k) + h_1 x_2(k) + T_s k_2 P_m; \)

**Step 3:** Proceeding to the last equation in Eq. (4), Similar to Step 2, we have
\[
H_2(k) = -M_1 x_1^2(k) - M_2 x_2^2(k) - [\sqrt{3} x_1(k) + \frac{c_1 - \sigma T_s}{\sqrt{3}} x_2(k)]^2 - \frac{(x_3(k) - x_3^*(k))^2}{2}
\]
\[
+ [(1 - \frac{T_s}{T_{qc}}) x_3(k) + \frac{T_s}{T_{qc}} u(k) + T_x \varepsilon_2(k) - \frac{n_{1(k+1)} F_2(k+1)}{1 + f(x_1(k+1))}]^2 - \frac{1}{2} \gamma^2 \varepsilon_2^2(k)
\]

where \( n_{1(k+1)} = n_{1(k)} \) \( \frac{1}{T_{e_{k3}} \sin (\delta_0 + x_1(k+1) + T_s x_2(k))} \); \( F_2(k+1) = c_1 x_1(k+1) + h_1 x_2(k+1) + k_2 P_m; \)

\( f(x_1(k+1)) = \frac{X_1 X_2}{\sqrt{h_2 + h_3 \cos (x_1(k+1) + \delta_0)}}; h_2 = (X_2 E_q^2 + (X_1 V_s)^2; h_3 = 2 X_1 X_2 E_q V_s \).

Assuming that the derivative of \( H_2(k) \) with respect to \( \varepsilon_2(k) \) is zero, then
\[
\varepsilon_2^*(k) = \frac{2 T_s^2}{2 T_s^2 - \gamma^2} F_3(k)
\]

where \( F_3(k) = -[(1 - \frac{T_s}{T_{qc}}) x_3(k) + \frac{T_s}{T_{qc}} u(k) - \frac{n_{1(k+1)} F_2(k+1)}{1 + f(x_1(k+1))}] + I_{q0}]. \)
Substituting Eq. (17) into Eq. (16), we have:

\[ H_2(k) = -M_1x_1^2(k) - M_2x_2^2(k) - [\sqrt{3}x_1(k) + \frac{\alpha - \sigma T}{\sqrt{3}}x_2(k)]^2 - [x_3(k) - x_3^*(k)]^2 + c_2F_3^2(k) \]  

(18)

The actual control input \( u(k) \) appears and our objective is to design \( u(k) \) so that \( e_1(k), e_2(k) \) and \( e_3(k) \) converge to zero. In order to make \( H_2(k) \leq 0 \), we finally choose the following feedback control \( u(k) \) by making \( F_3(k) = 0 \) as follows:

\[ u(k) = \frac{T_q}{T_s}F_4(k) + x_3(k) \]  

(19)

\[ F_4(k) = \frac{n_1(k+1)F_2(k+1)}{1 + f(x_1(k+1))} - I_{q0} - x_3(k) \]

Then we have \( H_2(k) = -M_1x_1^2(k) - M_2x_2^2(k) - (\sqrt{3}x_1(k) + \frac{\alpha - \sigma T}{\sqrt{3}}x_2(k))^2 - e_3^2(k) \leq 0 \). Let \( V(x(k)) = 2V_3(x(k)) \), therefore:

\[ \Delta V(x(k)) \leq \gamma^2\|\varepsilon(k)\|^2 - \|z(k)\|^2 \]  

(20)

We determine the summation of both sides of Eq. (23). The dissipation inequality can be obtained as: \( V(x(k)) - V(x(0)) \leq \sum_{k=0}^{N} (\gamma^2\|\varepsilon(k)\|^2 - \|z(k)\|^2) \). If \( \forall \gamma > 0 \), For any \( k > 0 \), the \( L_2 \) gain from the disturbance to the output of the system is smaller than or equal to disturbance attenuation constant \( \gamma \). In addition, \( H_2(k) \leq 0 \) is established for any interference.

The closed-loop error control system is asymptotically stable under the feedback control law \( u(k) \). It follows that \( e_1(k), e_2(k), e_3(k) \to 0 \) as \( k \to \infty \). According to the definition of virtual control, \( x_1(k), x_2(k), x_3(k) \) are bounded convergences.

### 3 Simulation Examples

This section presents a simulation analysis of the disturbance attenuation controller by considering the effect of the mechanical and short-circuit ground fault. The system physical parameters used in the simulation are as: \( D = 0.1 \text{ p.u.}, H = 7 \text{ s}, V_s = 0.995 \text{ p.u.}, E_q' = 1.7007 \text{ p.u.}, \omega_0 = 1 \text{ p.u.}, P_m = 0.9 \text{ p.u.}, X_0 = 0.3 \text{ p.u.}, X_T = 0.15 \text{ p.u.}, X_L = 0.0266 \text{ p.u.} \). The system parameters are designed as follows: \( \gamma = 1, q_1 = 0.6, q_2 = 0.4, T_s = 0.1, \sigma = 2 \).

Assuming that a relatively large mechanical power disturbance (approximately 10%) is added when \( t = 12 - 13 \text{ s} \), the simulation transient responses of states are shown in Figs. 1 and 2. For better analysis and understanding of the systems anti-interference ability, we introduced a 20% disturbances to the system. The simulation results are shown in Fig. 3.

Fig. 1 and Fig. 2 reveal that the convergence time of the sampled-data one is shorter, and the amplitude is smaller than those of the continuous one. Figs. 1 and Fig. 3 show that the dynamic response of the system does not change significantly with the increase in mechanical power disturbance. Therefore, the controller is not sensitive to the change in disturbance.

This part the transmission line ground fault is considered. we assume that the system is in a pre-fault steady state. We further suppose that a symmetrical transmission short-circuit ground fault occurs at \( t = 12 \text{ s} \). The faulted portion is isolated by the automatic opening of circuit breakers,
Fig. 1: Transient response with the proposed controller when 10% mechanical power disturbance occurs

Fig. 2: Transient response with continuous-time controller when 10% mechanical power disturbance occurs

Fig. 3: Transient response with the proposed controller when 20% mechanical power disturbance occurs

and the resistance of the transmission line changed after the fault. The dynamic responses of the system are shown in Figs. 4 and 5.

The proposed controllers are able to respond appropriately under different operating conditions. The power system shifts from an initial stable state to a new stable state without losing synchronization and operates safety under the new state despite having suffered from a large disturbance. This simulation example illustrates that for the digital control application, the proposed sampled-data stabilization controller demonstrates superior performance behavior.

4 Conclusions

This paper investigated a large-disturbances attenuation strategy for nonlinear STATCOM system. The intelligent adoption of the Minimax method enabled the design process to consider fully effects of the disturbances, and reduced conservativeness in dealing with disturbances. The scheme was validated by applying the digital controller to a continuous-time plant. When the operating situation changed, the dynamic response of the system not only quickly converged to a stable point and effectively inhibited external disturbances, but also become insensitive to the disturbance. The proposed sampled-data controller is proven to generate superior performances in digital control applications.

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References

Fig. 4: Transient response with the proposed controller when 10% mechanical power disturbance occurs

Fig. 5: Transient response with continuous-time controller when 10% mechanical power disturbance occurs


