Analysis on the Inter-carrier Interference of MIMO OFDM with Distributed Antenna Systems

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Abstract

The effect of Doppler spreading destroys the orthogonality of the subcarrier in OFDM giving rise to inter-carrier interference (ICI), and the Doppler spectrum types of MIMO with distributed antenna systems (DAS) may not be the same. This paper focuses on the analysis of ICI for MIMO OFDM with DAS. Firstly the instantaneous expression of SINR is presented using zero force (ZF) detector. Then the carrier to interference (C/I) ratio is derived to demonstrate the effect of the ICI under general Doppler spectrum. Numerical results show the correctness of the C/I ratio derived.

Keywords: Inter-carrier Interference; MIMO OFDM; Distributed Antenna Systems; Carrier to Interference (C/I) Ratio

1 Introduction

Orthogonal Frequency Division Multiplexing (OFDM) [1] transmission over wireless fading channels is very sensitive to time variations due to Doppler spreading. The effect of Doppler spreading destroys the orthogonality of the subcarriers, resulting in inter-carrier interference (ICI) due to power leakage among subcarriers. As a promising technique of future wireless communications, Multiple Input Multiple Output (MIMO) OFDM with Distributed Antenna Systems (DAS) have been attracting much research interest in the recent years [2]. In distributed MIMO OFDM, there are different types of Doppler spectrum giving rise different ICI.

In several previous publications [3] [4], the approximate expressions for the average signal-to-interference-plus-noise-ratio (SINR) of OFDM was analyzed based on the assumption that the ICI distribution is Gaussian by invoking the central limit theorem. In this regard, simple closed-form expressions and bounds for the total ICI power are given in [5]. In other related papers [6],

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effort has been made to evaluate the effect of ICI in MIMO OFDM systems. The authors of [7] have shown that the channel impulse response can be approximated by the first leading terms of its Taylor series and analyzed the carrier to interference (C/I) ratio. In [8] [9], Hamdi presents an exact SINR analysis of OFDM with multiple transmit antennas over doubly-selective fading channels, but the calculation of SINR is very complex due to the large-scale matrix computation and the integral calculation. On the other hand, in MIMO OFDM uplink with DAS, the received antennas are located at different places. Therefore, the Doppler spectrum type may be different for different received antennas. As there are different types of Doppler spectrum, a problem arises on how to determine the effect of ICI. As far as we know, there has been little concentration on this issue.

In this paper we focus on analyzing the ICI of MIMO OFDM with DAS in a frequency selective, Rayleigh fading time-varying channel. Firstly, the instantaneous expression of SINR is presented using zero force (ZF) detector [10]. Then, the C/I ratio is derived with the Doppler spectrum in a general sense. Finally, some numerical results are given to verify the correctness of the derivation.

2 System Model

2.1 Transmitter

The baseband OFDM signal can be expressed as

\[ x(t) = \frac{1}{\sqrt{N}} \sum_{p=0}^{N-1} X(p) e^{j2\pi \frac{p}{T_G}t}, t \in [-T_G, T], \]

where \( N \) is the number of subcarriers and \( X(p) \) is the \( M \)-ary complex information bearing symbol on the \( p \)th subcarrier, \( T_G \) is the length of the cyclic prefix (CP) and \( T \) is the (useful) OFDM symbol duration. In this paper, we assume that \( \mathbb{E}[X(p)] = 0 \) and \( \mathbb{E}[X(p)X(q)^*] = \delta(p-q)E_b, \forall p, q \), where \( \mathbb{E} \) is the expectation operator, \( \delta(\cdot) \) denotes the Dirac’s delta function, and \( E_b \) presents bit energy.

If \( 1/T \) is the symbol rate of the input data to be transmitted, the symbol interval in the OFDM system is increased to \( NT \), which makes the system more robust against the channel delay spread. Here, \( T = N/R \) where \( R \) is the symbol rate of the input data to be transmitted.

2.2 Channel

A WSSUS doubly-selective Rayleigh channel with the time-varying impulse response \( h(t, \tau) \) and a time-varying frequency response \( H(t, f) \) is considered in the following. The autocorrelation function \( R_H(t, f) \) of \( H(t, f) \) is assumed to be separable [7][11], i.e. it can be expressed as \( R_H(t, f) = R_D(t)R_H(f) \), where \( R_D(t) \) and \( R_H(f) \) are the temporal correlation function and the correlation in frequency across subcarriers, respectively.

The temporal correlation function of the classical (Jakes) Doppler power spectrum is modeled as [12]

\[ R_D(\tau) = J_0(2\pi f_D \tau), \quad \text{(Jakes)}. \]
where $f_D$ is the maximum Doppler frequency. Two extreme cases of the classical Doppler spectrum are the flat and the two-path models, which have been studied in [5] [12]. The corresponding temporal correlation functions for the two models are

$$R_D (\tau) = \text{sinc} (2f_D \tau), \quad \text{(flat)},$$

(3)

and

$$R_D (\tau) = \cos (2\pi f_D \tau), \quad \text{(two-path)},$$

(4)

where $\text{sinc} (x) \triangleq \sin (\pi x)/\pi x$. It should be noted that the two-path model corresponds to an OFDM system with a fixed frequency offset of $f_D$ Hz.

In order to specify the correlation in frequency across subcarriers, we adopt an exponential multipath power intensity of form

$$S_H (\tau) = \beta e^{-\beta \tau}, \quad \tau > 0, \beta > 0,$$

where $\beta$ is a parameter that controls the coherence bandwidth of the channel. The Fourier transform of $S_H (\tau)$ yields

$$R_H (k) = \frac{\beta}{\beta + j2\pi \Delta f k},$$

where $\Delta f = 1/T$ is the subcarrier spacing.

2.3 Receiver

The received signal after removing the CP is given by [9]

$$y (k) = \frac{1}{\sqrt{N}} \sum_{p=0}^{N-1} X (p) H \left( \frac{kT}{N}, p \right) e^{j \frac{2\pi}{N} pk} + z (k), k = 1, \cdots, N,$$

(6)

where $z(k)$ is the complex additive white Gaussian noise (AWGN) with zero-mean and two-sided power spectral density $N_0$.

Letting $Y(p)$, $X(p)$, and $Z(p)$ are the DFTs of \{y(k)\}, \{x(k)\} and \{z(k)\}, respectively, then the $N$-point FFT samples at the receiver are

$$Y (p) = \frac{1}{\sqrt{N}} \sum_{k=0}^{N-1} y (k) \exp \left( -j \frac{2\pi}{N} pk \right)$$

$$= G (p, p) X (p) \sum_{q=0}^{N-1} G (q, p) X (q) + Z (p), p = 0, \cdots, N-1,$$

(7)

where

$$G (q, p) = \frac{1}{N} \sum_{k=0}^{N-1} H \left( \frac{kT}{N}, q \right) e^{j \frac{2\pi}{N} (q-p)k},$$

(8)

and $Z(p)$ is a zero-mean complex Gaussian random variables (RVs) with variance $N_0$.

In the MIMO OFDM case [6], we can modify (7) to

$$Y (p) = G (p, p) X (p) + \sum_{q=0}^{N-1} G (q, p) X (q) + Z (p),$$

(9)
where the \((m, n)\)th \((1 \leq n \leq M_t, 1 \leq m \leq M_r)\) element of \(G(q, p)\) is

\[
[G(q, p)]_{m,n} = \frac{1}{N} \sum_{k=0}^{N-1} H^{(m,n)}(kT/q, q) e^{j\frac{2\pi}{N} (q-p)k}, \quad 0 \leq p \leq N - 1, \quad 0 \leq q \leq N - 1,
\]

where \(H^{(m,n)}\) denotes the \((m, n)\)th channel response in MIMO. In addition, the received vector for the \(p\)th frequency bin is \(Y(p) = [Y^{(1)}(p), \ldots, Y^{(M_r)}(p)]^T\), \(X(p) = [X^{(1)}(p), \ldots, X^{(M_r)}(p)]^T\) is the transmitted vector for the \(p\)th frequency bin, and \(Z(p) = [Z^{(1)}(p), \ldots, Z^{(M_r)}(p)]^T\) is the noise vector.

### 3 ICI Analysis

In this Section, we study the effort of Doppler spreading on ICI and the C/I ratio performance of MIMO OFDM with DAS using ZF detector over Rayleigh fading channel. In the receiver, the post-detection signal matrix with a linear ZF equalizer is

\[
G^\dagger(p, p) Y(p) = X(p) + G^\dagger(p, p) [I(p) + Z(p)],
\]

where \((\cdot)^\dagger\) denotes pseudo-inverse, \(I(p) \triangleq \sum_{q \neq p} G(q, p) X(q)\). Then the post-detection SINR with a linear ZF equalizer of the \(n\)th \((1 \leq n \leq M_t)\) transmit antenna at the \(p\)th \((0 \leq p \leq N - 1)\) subcarrier is [10]

\[
\gamma_{p,n} = \frac{[R_X]_{nn}}{\left[\sum_{q \neq p} G^\dagger(q, p) G(q, p) \right]_{nn}}^{1/2},
\]

where \(R_X \triangleq E[XX^H(p)] = E_b/M_t \ I_{M_t}, \ R_Z \triangleq E[ZZ^H(p)] = N_0 I_{M_r}\), and \(R_{ICI} \triangleq E[I(p)I^H(p)]\). The variance matrix of the ICI term is

\[
R_{ICI} = E[I(p)I^H(p)]
\]

\[
= E \left[ \sum_{q_1=0}^{N-1} \sum_{q_2=0}^{N-1} \sum_{q_1 \neq q_2}^{N-1} G(q_1, p) X(q_1) X^H(q_2) G^H(q_2, p) \right]
\]

\[
= \sum_{q_1=0}^{N-1} \sum_{q_2=0}^{N-1} E \left[ G(q_1, p) X(q_1) X^H(q_2) G^H(q_2, p) \right]
\]

\[
\sum_{q=0}^{N-1} E \left[ G(q, p) X(q) X^H(q) G^H(q, p) \right].
\]

Note that \((G(q_1, p), G^H(q_2, p))\) is independent of \((X(q_1), X^H(q_2))\). Also, the \(X(q)\)’s are i.i.d.
with zero means. Thus $J_1 = 0$. It then follows that

$$
\mathbf{R}_{\text{ICI}} = \sum_{q=0}^{N-1} \mathbb{E} \left[ \mathbf{X}(q) \mathbf{X}^H(q) \right] \mathbb{E} \left[ \mathbf{G}(q,p) \mathbf{G}^H(q,p) \right] = \frac{E_b}{M_t} \sum_{q=0}^{N-1} \mathbb{E} \left[ \mathbf{G}(q,p) \mathbf{G}^H(q,p) \right].
$$

(14)

The matrix $\mathbf{G}(q,p)$ has i.i.d. complex Gaussian elements, then

$$
\mathbf{R}_{\text{ICI}} = \mathbb{E} \left[ \mathbf{G}^H(p,p) \left( \mathbf{R}_{\text{ICI}} + \mathbf{R}_Z \right)^{-1} \mathbf{G}(p,p) \right],
$$

and the corresponding expectation matrix is $\mathbf{\Sigma} = \mathbb{E} \left[ \mathbf{W} \right]$, then the average SINR is [10]

$$
\bar{\gamma}_{p;nn} = \frac{E_b/M_t}{\mathbb{E} \left[ \mathbf{G}^H(p,p) \mathbf{G}(p,p) \right] - \mathbb{E} \left[ \mathbf{G}^H(p,p) \mathbf{G}(p,p) \right]},
$$

(16)

In the following, we derive the expression of $\bar{\gamma}_{p;nn}$. Define $\mathbf{\Lambda} \equiv \text{diag} (\lambda_1, \cdots, \lambda_{M_r}) \triangleq (\mathbf{R}_{\text{ICI}} + \mathbf{R}_Z)^{-1}$, where diag is a diagonal matrix. Then, we have

$$
\mathbf{\Sigma} = \mathbb{E} \left[ \mathbf{G}^H(p,p) \mathbf{\Lambda} \mathbf{G}(p,p) \right] = \text{diag} \left( \sum_{m=1}^{M_r} \lambda_m \mathbb{E} \left[ |G^{(m,1)}(p,p)|^2 \right], \cdots, \sum_{m=1}^{M_r} \lambda_m \mathbb{E} \left[ |G^{(m,M_t)}(p,p)|^2 \right] \right).
$$

(17)

As

$$
[\mathbf{R}_{\text{ICI}}]_{mm} = \frac{E_b}{M_t} \sum_{q=0}^{N-1} \sum_{n=1}^{M_t} \mathbb{E} \left[ |G^{(m,n)}(q,p)|^2 \right],
$$

(18)

and $[\mathbf{R}_Z]_{mm} = N_0$, $1 \leq m \leq M_r$, then, for $p = 0, \cdots, N - 1, 1 \leq n \leq M_t$, from (16) and (17) we have

$$
\bar{\gamma}_{p;nn} = \frac{E_b}{M_t} \sum_{m=1}^{M_r} \lambda_m \mathbb{E} \left[ |G^{(m,n)}(p,p)|^2 \right] = \sum_{m=1}^{M_r} \left( \sum_{q=0}^{N-1} \sum_{n=1}^{M_t} \mathbb{E} \left[ |G^{(m,n)}(q,p)|^2 \right] \right) + \frac{M_t}{\text{SNR}},
$$

(19)
where SNR = $E_b/N_0$. On the other hand, we have

$$
\mathbb{E} \left[ |G^{(m,n)}(q,p)|^2 \right] = \frac{1}{N^2} \sum_{k_1=0}^{N-1} \sum_{k_2=0}^{N-1} \mathbb{E} \left[ H^{(m,n)} \left( \frac{k_1 T}{N}, q \right) \left( H^{(m,n)} \left( \frac{k_2 T}{N}, q \right) \right) \right] e^{j \frac{2\pi}{N} (q-p)(k_1-k_2)}
$$

$$
= \frac{1}{N^2} \sum_{k_1=0}^{N-1} \sum_{k_2=0}^{N-1} R_D^{(m)} \left( \frac{k_1 - k_2}{T} \right) R_H(0) e^{j \frac{2\pi}{N} (q-p)(k_1-k_2)}
$$

$$
= \frac{1}{N^2} \left[ N + \sum_{k=1}^{N-1} (N-k) R_D^{(m)} \left( \frac{kT}{N} \right) \left( e^{j \frac{2\pi}{N} (q-p)k} + e^{-j \frac{2\pi}{N} (q-p)k} \right) \right], \quad (20)
$$

where $R_D^{(m)}(\cdot)$ denotes the temporal correlation function $R_D(\cdot)$ of the $m$th ($1 \leq m \leq M_r$) received antenna, and is some one model of Jakes’ spectrum, flat spectrum, and two-path spectrum. Note that for the different receivers, $R_D^{(m)}(\cdot)$ may be different. We demonstrate the effect of ICI under general Doppler spectrum. $\mathbb{E} \left[ |G^{(m,n)}(q,p)|^2 \right]$ are seen to be identical for all $1 \leq n \leq M_t$, then we have

$$
\sum_{q\neq p} \sum_{n=1}^{M_t} \mathbb{E} \left[ |G^{(m,n)}(q,p)|^2 \right] = \sum_{q\neq p} \frac{M_t}{N^2} \left[ N + \sum_{k=1}^{N-1} (N-k) R_D^{(m)} \left( \frac{kT}{N} \right) \left( e^{j \frac{2\pi}{N} (q-p)k} + e^{-j \frac{2\pi}{N} (q-p)k} \right) \right]
$$

$$
= M_t \frac{1}{N^2} \left\{ N (N-1) + \sum_{k=1}^{N-1} (N-k) R_D^{(m)} \left( \frac{kT}{N} \right) \left[ \sum_{q=0}^{N-1} \left( e^{j \frac{2\pi}{N} (q-p)k} + e^{-j \frac{2\pi}{N} (q-p)k} \right) - 2 \right] \right\}
$$

$$
= M_t \frac{1}{N^2} \left\{ N (N-1) - 2 \sum_{k=1}^{N-1} (N-k) R_D^{(m)} \left( \frac{kT}{N} \right) \right\}, \quad (21)
$$

and

$$
\mathbb{E} \left[ |G^{(m,n)}(p,p)|^2 \right] = \frac{1}{N^2} \left[ N + 2 \sum_{k=1}^{N-1} (N-k) R_D^{(m)} \left( \frac{kT}{N} \right) \right]. \quad (22)
$$

Hence, from (19)-(22), the expression of $\tilde{g}_{p,n}$ can be written as

$$
\tilde{g}_{p,n} = \frac{1}{M_t} \sum_{m=1}^{M_t} \frac{1}{N^2} \left\{ N (N-1) - 2 \sum_{k=1}^{N-1} (N-k) R_D^{(m)} \left( \frac{kT}{N} \right) \right\} + \frac{1}{\text{SNR}}, \quad (23)
$$

and the C/I ratio is

$$
\frac{C}{I} = \frac{1}{M_t} \sum_{m=1}^{M_t} \frac{N + 2 \sum_{k=1}^{N-1} (N-k) R_D^{(m)} \left( \frac{kT}{N} \right)}{N (N-1) - 2 \sum_{k=1}^{N-1} (N-k) R_D^{(m)} \left( \frac{kT}{N} \right)}. \quad (24)
$$
4 Numerical Results

In this section, we compare the C/I ratio of distributed MIMO OFDM with the C/I ratio of MIMO OFDM without using DAS. We choose $M_t = 2$ and $M_r = 2$. The OFDM system is assumed to have $N = 256$ subcarrier, with subcarrier spacing $\Delta f = 7.81$ KHz and carrier frequency $f_c = 2$ GHz.

![C/I ratio curves of distributed MIMO OFDM with same Doppler spectrum types.](image1)

The C/I ratio curves of distributed MIMO OFDM with same Doppler spectrum types are plotted versus the maximum Doppler frequency in Fig. 1. For all $m (m = 1, \cdots, M_r)$, $R^{(m)}_D (\cdot)$ are assumed to be flat model, Jakes’ model, and two-path model, respectively. From Fig. 1 we can see that, the C/I ratio curves derived match very well with the results given in [5][7]. Furthermore, the C/I ratio of flat spectrum is higher than that of Jakes’ spectrum, and the C/I ratio of two-path spectrum is lower than that of Jakes’ spectrum.

![C/I ratio curves of distributed MIMO OFDM with different Doppler spectrum types.](image2)

Fig. 2: C/I ratio curves of distributed MIMO OFDM with different Doppler spectrum types.
The C/I ratio curves of distributed MIMO OFDM with different Doppler spectrums are plotted versus the maximum Doppler frequency in Fig. 2. From Fig. 2 we can see that, when $R_D^{(1)}(\cdot)$ is assumed to be Jakes’ model and $R_D^{(2)}(\cdot)$ is assumed to be flat model, the C/I ratio curve is higher than that of pure Jakes’ model, and lower than that of pure flat model. When $R_D^{(1)}(\cdot)$ is assumed to be Jakes’ model and $R_D^{(2)}(\cdot)$ is assumed to be two-path model, the C/I ratio curve is higher than that of pure two-path model, and lower than that of pure Jakes’ model. This phenomenon is due to that, in DAS the received antennas are located at different places, and the Doppler spectrum types may be different for different receiver.

5 Conclusions

This letter analyzes the ICI and presents a C/I ratio expression for MIMO OFDM with distributed antennas systems. Numerical results show the correctness of the C/I ratio derived. Furthermore, the C/I ratio of MIMO OFDM with different Doppler spectrum types is no longer the same with the C/I ratio of MIMO OFDM with one Doppler spectrum type.

References


