An Classification-based Adaptive Decision Feedback Equalizer for Rayleigh Multipath Channel* 

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Abstract

This paper aims at dealing with the issue of adaptive equalization for Rayleigh multipath fading channel. A classification-based adaptive decision feedback equalizer (DFE) using recursive least squares (RLS) algorithm is proposed in this paper. Unlike the standard DFE, the classification-based DFE considers the decision device of the DFE as a classifier from the point of view of classification. Given the superior performance of the support vector machine (SVM) classification algorithm, we use an SVM classifier as the decision device of the DFE to improve the accuracy of signal decision. Also the performance of the SVM-based DFE is compared with that of the standard DFE using adaptive RLS algorithm. The simulation results illustrate that the SVM DFE could achieve much lower bit error rate (BER) at the receiver in Rayleigh multipath fading channel than the standard DFE using adaptive RLS algorithm.

Keywords: Decision Feedback Equalizer; SVM; Recursive Least Square; Rayleigh Multipath Channel; Classification

1 Introduction

In digital wireless communication systems, high bit rate transmission over multipath fading channels always suffer from a type of distortion known as inter symbol interference (ISI) which will dramatically increase bit error rate (BER) at the receiver and reduce the system performance [1]. In order to suppress the ISI and improve the reliability of digital wireless communication systems, the adaptive channel equalization is developed for combating ISI.

It is known that decision feedback equalizer (DFE) is the most popular nonlinear equalizer for severe fading channels [2]. A DFE is usually composed of a feed forward filter (FFF), a feedback filter (FBF) and a decision device. Unlike linear equalizers, DFE uses both received symbols

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and previously detected symbols to process currently received symbols. The previously detected symbols are fed back through the FBF as the input of the FFF to cancel their interference on currently received symbols which is exactly the ISI. Also the DFE could track a time-variant fading channel by using adaptive algorithms to update its weight vector according to equalization errors. There are two well-known adaptive algorithms called as recursive least squares (RLS) and least mean squares (LMS) algorithms which are widely used for decision feedback equalization. Though LMS algorithm is simple and executes quickly, its slow convergence nature make it useful only for slowly varying channels [3]. However, the RLS algorithm has faster convergence rate and better tracking ability than LMS [4] at the cost of increasing a little computational complexity and its convergence speed is independent of the eigen-value spread [5]. Therefore, the RLS algorithm is adopted for DFE design in the paper.

Because the previously detected symbols are fed back into DFE, it is obvious that the performance of the DFE is significantly affected by the effectiveness of its decision device. Consequently, a well-designed decision device will dramatically improve the reliability of the DFE. In this paper, from the point of view of classification, we find that the DFE’s decision device could be regarded as a classifier if each modulated signal (e.g., QPSK modulated signal) value is considered as a category. Moreover, the number of categories depends on the digital modulation type of the transmitted digital signals. Therefore, we could consider that the nearest neighbor (NN) classifier is adopted as the decision device for the standard DFE. Moreover, it is reasonable for us to believe that the reliability and effectiveness of the standard DFE could be improved by choosing a superior classifier. As well known, SVM is a very famous algorithm for classification and it has been widely used in classification, pattern recognition and other applications for its characteristics of rapid convergence and classification accuracy [6]. Given the superior performance of SVM-based classifiers, we use an SVM-based classifier as the decision device of the DFE for Rayleigh multipath fading channel equalization.

The rest of this paper is organized as follows: Section 2 discusses the problem of decision feedback equalization in digital communication systems and an SVM-based equalization algorithm is proposed. The performance of the SVM-based DFE is analyzed and compared with that of the standard DFE by simulation in Section 3. Finally, Section 4 gives the conclusions.

2 SVM-based Decision Feedback Equalization

The DFE has been widely used to combat ISI in digital communication systems especially in situations where the channel distortion is severe [7]. In the paper, it is assumed that digital signals are transmitted over Rayleigh multipath fading channel and the block diagram of a simplified digital communication system is shown as Fig. 1. Digital signals from the data source are modulated (e.g., using binary phase shift keying (BPSK) or quadrature phase shift keying (QPSK)) and filtered with upsampling by a pulse shaping filter at the transmitter to generate the baseband waveform which is then transmitted over the channel with carrier modulation ignored. The Rayleigh multipath fading channel here is modeled by the method proposed in [8]. At the receiver, the signals passing through the Rayleigh multipath fading channel added with the noise \( v(t) \) are filtered using the same pulse shaping filter with downsampling and fed into a DFE. At last, the symbols are demodulated and sent to the data sink after equalization using the DFE.

Generally, the DFE could adaptively adjust its weight vector \( \mathbf{W}(n) \) to minimize the error \( e(n) \) based on a certain criterion in real-time and Fig. 2 shows the block diagram of an SVM-based
DFE. As the standard DFE, it also has two operation modes called the training mode and tracking mode. In the training mode, it switches to $A$ and the training sequence $d(n)$ is used to get the optimum weight vector $\mathbf{W}(n)$ at time $n$ and the optimum parameters for the SVM classifier. At the end of the training mode, the SVM-based DFE starts to operate in the tracking mode by switching to $B$. Let $m$ and $k$ denote the orders of the feed-forward filter (FFF) and feedback filter (FBF) respectively, then in the tracking mode the weight vector $\hat{\mathbf{W}}(n)$ could be written as Eq. (1).

\[
\hat{\mathbf{W}}(n) = \left[ \mathbf{W}_{FF}(n), \mathbf{W}_{FBF}(n) \right]^T, \tag{1}
\]

where $\mathbf{W}_{FF}(n)$ and $\mathbf{W}_{FBF}(n)$ are the weight vectors of the FFF and FBF at time $n$ respectively.

And the input vector $\hat{\mathbf{X}}(n)$ is defined in Eq.(2).

\[
\hat{\mathbf{X}}(n) = \left[ x(n) \cdots x(n-m+1), \hat{d}(n-1) \cdots \hat{d}(n-k+1) \right]^T, \tag{2}
\]

where $x(n)$ represents the input sample of the DFE and $\hat{d}(n)$ denotes the output sample of the decision device at time $n$. Therefore, the output of the DFE in the tracking mode could be calculated as follows.

\[
\hat{\mathbf{y}}(n) = \mathbf{W}^T(n)\hat{\mathbf{X}}(n), \tag{3}
\]

where $\hat{\mathbf{y}}(n)$ is the output sample of the DFE at time $n$. The difference between $\hat{d}(n)$ and $\hat{\mathbf{y}}(n)$ called the decision error $\hat{e}(n)$ is presented in Eq. (4).

\[
\hat{e}(n) = \hat{d}(n) - \hat{\mathbf{y}}(n), \tag{4}
\]

Eqs. (1-4) above show the filtering procedure of the DFE. It could be found that the output of the decision device significantly affect the reliability of the DFE according to Eq. (2) because $\hat{d}(n)$ is a part of the input of the DFE.
Table 1: SVM-based DFE using RLS algorithm

**Training Mode:**

1) Initialization

The weight vector $\mathbf{W}$,

$$\mathbf{W}(-1) = [0 \ 0 \ldots 0]^T$$

The inverse of the input autocorrelation matrix $\mathbf{P}$,

$$\mathbf{P}(-1) = \sigma \mathbf{I}, 0 < \delta \leq 1$$

Where $\sigma$ may be the inverse of the input signal power estimation.

2) Do for $n \geq 0$

Repeat the following steps to get the optimum weight vector $\mathbf{W}_{opt}$ and $\lambda$ is the forgetting factor.

$$e(n) = d(n) - \mathbf{X}^T(n)\mathbf{W}(n-1)$$

$$\mathbf{P}(n) = \frac{1}{\lambda} \left( \mathbf{P}(n-1) - \frac{\mathbf{P}(n-1)\mathbf{X}(n)\mathbf{X}^T(n)\mathbf{P}(n-1)}{\lambda + \mathbf{X}^T(n)\mathbf{P}(n-1)\mathbf{X}(n)} \right)$$

$$\mathbf{W}(n) = \mathbf{W}(n-1) + \mathbf{P}(n)\mathbf{X}(n)e(n)$$

3) Output

If necessary compute

$$y(n) = \mathbf{W}^T(n)\mathbf{X}(n)$$

**Tracking Mode:**

1) Initialization

$$\hat{\mathbf{W}}(-1) = \mathbf{W}(N)$$

$$\hat{\mathbf{P}}(-1) = \mathbf{P}(N)$$

where $N$ is the length of the training sequence and $\mathbf{W}(N)$ is the optimum weight vector got in the training mode. $\mathbf{P}(N)$ is the inverse of the input autocorrelation matrix at time $N$.

2) Do for $n \geq 0$

Repeat the following steps to update the weight vector $\hat{\mathbf{W}}(n)$ for tracking.

$$\hat{e}(n) = \hat{d}(n) - \mathbf{X}^T(n)\hat{\mathbf{W}}(n-1)$$

$$\hat{\mathbf{P}}(n) = \frac{1}{\lambda} \left( \hat{\mathbf{P}}(n-1) - \frac{\hat{\mathbf{P}}(n-1)\mathbf{X}(n)\mathbf{X}^T(n)\hat{\mathbf{P}}(n-1)}{\lambda + \mathbf{X}^T(n)\hat{\mathbf{P}}(n-1)\mathbf{X}(n)} \right)$$

$$\hat{\mathbf{W}}(n) = \hat{\mathbf{W}}(n-1) + \hat{\mathbf{P}}(n)\mathbf{X}(n)\hat{e}(n)$$

where $\hat{d}(n)$ is the output of the SVM-based decision device at time $n$.

3) Output

If necessary compute

$$\hat{y}(n) = \hat{\mathbf{W}}^T(n)\mathbf{X}(n)$$
Since the baseband signals are modulated with some kind of modulation technique (e.g., BPSK, QPSK) in digital communication systems, they are fixed at several values. And the objective of the decision feedback equalization is to restore the transmitted baseband signals from the received signals. Therefore, we can design the decision device of the DFE from the point of view of classification. Each modulation value may be regarded as a category and the number of categories depends on the modulation type. For example, choosing the QPSK modulation, the number of categories is 4 and the role of the decision device is to divide its input signal sequence $y(n)$ into 4 categories. Consequently, we could also call the decision device as classifier and it is easily found that the standard DFE uses the NN-based classifier as its decision device. Here, given the superior performance of SVM-based classifiers, we use the standard SVM classifier as the DFE’s decision device for channel equalization. Moreover, adaptive algorithms based on certain criterions are often adopted to adaptively adjust the weight vector of the DFE and minimize the decision error. Here, we choose the standard RLS algorithm based on the least squares criterion to solve the decision feedback equalization problem described above. The computational steps in Tab.1 describe the standard RLS algorithm as applied to the DFE with an SVM classifier as its decision device which we call it as the SVM-based DFE.

3 Simulation Experiments

In this section, the simplified digital communication system as shown in Fig. 1 is simulated to examine the BER performance of the SVM-based DFE using the RLS algorithm and the simulation results are also compared with that of the standard DFE using the same algorithm. Let $\lambda$ denote the forgetting factor of the standard RLS algorithm, $m$ and $k$ stand for the orders of the FFF and FBF respectively and $f_d$ represent the maximum Doppler shift of a three-path Rayleigh frequency selective fading channel. The values of these public parameters are set in Tab. 2. And the well-known toolbox called libsvm in [9] is employed to construct an SVM classifier with the default parameters provided in libsvm.

<table>
<thead>
<tr>
<th>Scenario</th>
<th>$\lambda$</th>
<th>$m$</th>
<th>$k$</th>
<th>Modulation Type</th>
<th>$f_d$(HZ)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.99</td>
<td>16</td>
<td>5</td>
<td>BPSK</td>
<td>10</td>
</tr>
<tr>
<td>2</td>
<td>0.99</td>
<td>16</td>
<td>5</td>
<td>QPSK</td>
<td>10</td>
</tr>
</tbody>
</table>

Fig. 3 and Fig. 4 show the simulation results of the scenario 1 in Tab. 2. Fig. 3 is the scatter plot of the BPSK signals at the receiver and it qualitatively illustrates that the SVM-based DFE has much better equalization performance than the standard DFE does using the RLS algorithm. Moreover, the BER performance comparison of both DFE’s shown in Fig. 4 quantitatively verifies that the SVM-based DFE does perform much better. Moreover, the simulation results of the scenario 2 in Tab.2 are presented in Fig. 5 and Fig. 6 which also indicate that the SVM-base DFE performs better than the standard DFE does using the RLS algorithm. Here, the learning curves of both DFE’s are not given because they are the same as the standard DFE using the RLS algorithm which means that the SVM-based DFE has no advantage in convergence rate.
Fig. 3: The scatter plot of the BPSK modulated signals transmitted over a three-path Rayleigh frequency selective fading channel with a maximum Doppler shift $f_d$ of 10HZ.

Fig. 4: BER performance comparison of both DFE’s for a three-path Rayleigh frequency selective fading channel with BPSK modulation and a maximum Doppler shift $f_d$ of 10HZ.

Fig. 5: The scatter plot of the QPSK modulated signals transmitted over a three-path Rayleigh frequency selective fading channel with a maximum Doppler shift $f_d$ of 10HZ.
Fig. 6: BER performance comparison of both DFE’s for a three-path Rayleigh frequency selective fading channel with QPSK modulation and a maximum Doppler shift $f_d$ of 10HZ.

4 Conclusions

This paper proposes an SVM-based DFE from the point of view of classification. The performance of the SVM-based DFE using the RLS algorithm is analyzed and the simulation results in different scenarios are presented. Compared with the standard DFE with the same algorithm, it is illustrated that the SVM-based DFE could achieve much better BER performance. However, the SVM-base DFE has no advantage in convergence rate and the computational complexity is increased because the SVM classifier is used. Therefore, our future work will focus on reducing the computational complexity and increasing the convergence rate.

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References


