Integrated Qualitative Spatial Reasoning and Its Application

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Abstract

Qualitative spatial reasoning is an important analysis method for spatial information in GIS and other applications. Recently, the integration of two spatial relations in a model is a hot topic in qualitative spatial reasoning. The combinations of more than three relations and multi-types of objects are very useful for solving practical problems, but neither of them has been studied before. Firstly, the general framework for multi-relations and multi-domains model is defined, so does the algorithms for solving the constraints satisfaction problem. Then a concrete model (PLRTSD) with three domains (Point, Line and Region) and three spatial relations (Topology, Size and Diameter) is studied. Finally, the PLRTSD model is applied to complex spatial queries. Powerful spatial queries can be built upon PLRTSD and the query execution time can be greatly reduced by our algorithms.

Keywords: Qualitative Spatial Reasoning; Qualitative Constraints Satisfaction Problem; Multi-domains spatial objects; Combining Multi-Relations

1 Introduction

Qualitative spatial representation and reasoning are known as the Qualitative Spatial Reasoning (QSR) in Artificial Intelligence [1]. As a high level processing technology for spatial data, QSR is widely used in robot navigation, GIS, video/image understanding, qualitative simulation and common sense reasoning. The reasoning task in QSR is a qualitative constraints satisfaction problem (QCSP). The basic function of QCSP is to answer the question whether the qualitative spatial constraints described by one or more spatial relational model can be satisfied.

In QSR, combining two different spatial relations on the same set of variables, known as the joint satisfaction problem (JSP), received extensive attentions. However, combining three or more spatial relations has not been studied before. Further more, integrating multi-types of objects are very common in GIS and other applications, but has not been investigated in QSR before. The...
The problem of integrated qualitative spatial reasoning, which studies the combination of more than three relations and multi-types of objects (named multi-domains here), is investigated in this paper. This problem comes from GIS and other applications, but has not been studied before.

The paper is organized as follows: In section 2 we review some related works. The general framework of integrated qualitative spatial reasoning is given in section 3. The PLRTSD model and its application are discussed in section 4 and 5 as a concrete case of the general framework. Section 6 draws some conclusions.

2 Related Works

In QSR, Spatial object can be point, line, closed rectangle, closed disk, closed region etc. The spatial configuration is depicted by the relations between the objects, and the relations are mostly binary. Dozens of spatial (temporal) relational models are proposed, such as Region Connection Calculus (RCC) [2]. They capture different types of binary spatio-temporal relations. Combinations of two spatial relations were investigated before. Gerevini and Renz study the JSP of RCC and size [3]. They investigate the combination of qualitative size (QS) and RCC8. The algorithm bipath-consistency is proposed. The JSP of two basic relation-constraint networks is bipath-consistency if the two separate constraint networks are all algebraically closed. And for each single pair of variables the two basic relation constraints on them are not disjointed. For the JSP of QS and basic RCC8, bipath-consistency determines consistency. The combination of RCC and metric size constraints is also studied in [3]. Li et al. investigate the JSP of RCC and direction by means of constructional algorithm and algebraic deduction [4, 5]. Wolff and Westphal divide JSP methods into tight combination and loose combination, and give some common principles of JSP [6]. Li et al. study whether the combination of two consistent networks shared a small group of common variables is still consistent. They provide the sufficient conditions for two basic relation networks sharing only one edge [7]. Combining more than three spatial relations or multi-types of object has not been studied before.

3 General Framework

Multi-domains may be encountered in many applications. For instance, in a world map, the capital layer consists of points which stand for the capitals, the river layer consists of lines for the rivers and the lake layer consists of regions for the lakes. Here we formally define the spatial relation integration on multi-domains.

Definition 1 $OD_i(1 \leq i \leq m)$ is an object domain i.e. a type of objects like points, lines, circles or rectangles. $R$ is a multi-domain relation on $OD_1, OD_2, ..., OD_m$, if

1. $OD_i \cap OD_j = \phi(i \neq j)$.
2. For every basic relation $r \in R$, $\exists OD_i, OD_j \in \{OD_1, OD_2, ..., OD_m\}, r \subseteq OD_i \times OD_j$.

Definition 2 $R$ is jointly exhaustive on $(OD_i, OD_j)$ if $\bigcup \{r | \forall r \in R, \text{Domain}(r) = (OD_i, OD_j)\} = OD_i \times OD_j$. $R$ is jointly exhaustive on $\{OD_1, OD_2, ..., OD_m\}$ if for $\forall(OD_i, OD_j) \in \{OD_1, OD_2, ..., OD_m\}(1 \leq i, j \leq m)$, $R$ is jointly exhaustive on $(OD_i, OD_j)$. 
**Definition 3** \( R \) is a basic JEPD relation on \( \{OD_1, OD_2, ..., OD_m\} \) if

1. \( R \) is pairwise disjoint i.e. \( r_1, r_2 \in R \) and \( r_1 \neq r_2 \rightarrow r_1 \cap r_2 = \phi \);
2. \( R \) is jointly exhaustive on \( \{OD_1, OD_2, ..., OD_m\} \);
3. The identity relations \((x, x)\) is contained in \( R \);
4. \( R \) is closed under converses.

**Definition 4** Given \( r, s \in R \), weak composition is defined as follows[8]:

\[
 r \circ s = \{ t | \exists a \in OD, \exists b \in OD, \exists c \in OD: arb \text{ and } bsc \text{ and } atc \}
\]

The weak composition on the multi-domain spatial relation set \( R \) must satisfy:

\[
 \text{domain}(r_1) = (OD_a, OD_b), \text{domain}(r_2) = (OD_c, OD_d), \text{ then } OD_b \neq OD_c \rightarrow r_1 \circ r_2 = \phi.
\]

**Definition 5** \( V \) is a set of variables on multi-domains \( OD = \{OD_1, OD_2, ..., OD_m\} \). Given \( p \) constraints set \( \nabla_k, (1 \leq k \leq p) \) on \( V \) where \( \nabla_k \) is the constraint based on spatial relational model \( R_k \). These \( p \) spatial relational models are different from each other, and they are all defined on \( OD \times OD \). Then \( MMCSP(V, \bigcup_{1 \leq k \leq p}\nabla_k) \) is the qualitative constraint satisfaction problem [1] of multi-domains multi-spatial-relations model. \( MMCSP(V, \bigcup_{1 \leq k \leq p}\nabla_k) \) is consistent iff there is a solution (instance) of \( V \) on \( OD \) satisfying \( \bigcup_{1 \leq k \leq p}\nabla_k \).

**Definition 6** \( MMCSP(V, \bigcup_{1 \leq k \leq p}\nabla_k) \) is multi-algebraically-closed iff

1. \( \nabla_k \) is algebraically closed \( 1 \leq k \leq p \) (i.e. every three variables satisfying weak composition [8]);
2. For each pair of variables \((x_i, x_j) \in V, x_i \sigma^k_{ij} x_j \in \nabla_k, \text{Dependency}(\bigcap_{1 \leq k \leq p} \sigma^k_{ij}) = \text{true} \) iff
   \[
   \bigcap_{1 \leq k \leq p} \sigma^k_{ij} \neq \phi \text{ (i.e there are no conflicts between these relations.)}
   \]
   bipath-consistency [3] is a special case of multi-algebraically-closed. By improving the bipath-consistency algorithm [3], we give a similar algorithm for MMCSP, the multi-algebraically-closed algorithm.

**Algorithm:** Multi-algebraically-closed\((V, \bigcup_{1 \leq k \leq p}\nabla_k)\)

Input: \( MMCSP(V, \bigcup_{1 \leq k \leq p}\nabla_k) \)

Output: return false, if \( MMCSP(V, \bigcup_{1 \leq k \leq p}\nabla_k) \) is not multi-algebraically-closed; otherwise, return true.

1. \( Q \leftarrow \{(i, j)|x_i, x_j \in V \text{ and } i < j \} \)
2. while \( Q \neq \phi \) do
3. {  
4. select and delete a pair \((i, j)\) from \( Q \);
5. for \( k \neq i, k \neq j, x_k \in V \) do
6. { 
7. if not Revision(i, j, k) return false; 
8. if not Revision(k, i, j) return false; 
9. } 
10. } 
11. return true. 

Function: Revision(i, k, j) 
Output: When weak composition as well as the dependency of the different kind of relations 
are considered together: if empty relation encountered, return false; otherwise return true. 
1. changed ← false; 
2. for s ← 1 to p 
3. \( o_s \leftarrow r^s_{ij} \); // \( x_i r^s_{ij} x_j \) is the \( R_s \) constraint on \( x_i, x_j \) in \( \nabla_s \). 
4. if not (Corelation(i, j) and Corelation(i, k) and Corelation(k, j)) then return false; 
5. for s ← 1 to p 
6. { 
7. \( r^s_{ij} \leftarrow r^s_{ij} \cap (r^s_{ik} \circ r^s_{kj}) \); //weak composition on \( R_s \) 
8. if \( o_s \neq r^s_{ij} \) then changed ← true; 
9. } 
10. if (not changed) then return true; 
11. if \( (i, j) \notin Q \) then add \( (i, j) \) to \( Q \); 
12. If not Corelation(i, j) then return false; 
13. for s ← 1 to p 
14. if \( r^s_{ij} \neq o_s \) then \( r^s_{ij} \leftarrow \text{reverse}(r^s_{ij}) \); 
15. return true. 

Function: Corelation(i,j) 
Output: When relation dependency on one pair of variables is considered: returns false, if 
empty relation encountered; otherwise return true. 
1. \( W \leftarrow \{(\theta_1, \theta_2, ..., \theta_p)|\theta_t \text{ is a basic relation in } r^t_{ij}(1 \leq t \leq p) \text{ and } \text{Dependency}(1, 2, ..., p) = \text{true}\}\}; 
2. if \( W = \phi \) then return false; 
3. for s ← 1 to p 
4. { 
5. \( t_s \leftarrow \lor\{\theta_s|(..., \theta_s, ...) \in W\} \}; 
6. if \( r^s_{ij} \neq t_s \) then 
7. { 
8. \( r^s_{ij} \leftarrow t_s \); 
9. if \( (i, j) \notin Q \) then add \( (i, j) \) to \( Q \);
10. } 
11. } 
12. return true.

**Theorem 1** $\text{MMCSP}(V, \bigcup_{1 \leq k \leq p} \nabla_k)$ is multi-algebraically-closed iff the algorithm $\text{Multi-algebraically-closed}(V, \bigcup_{1 \leq k \leq p} \nabla_k)$ returns true.

Proof. $\text{Revision}(i, k, j)$ ensures that every $\nabla_k$ are algebraically closed, while $\text{Corelation}(i, j)$ guarantees $\bigcap_{1 \leq k \leq p} \sigma_{ij}^k \neq \emptyset$ because at least one basic relation combination is feasible ($\text{Dependency}(\theta_1, \theta_2, ..., \theta_p) = \text{true}$).

### 4 A Demonstration: PLRTSD Model

Based on the general framework for multi-relations and multi-domains model, we give a concrete case in this section, the PLRTSD model. Three domains (Points, Lines and Regions) and three spatial relations (Topology, Size and Diameter) are integrated in the PLRTSD. Three object domains are considered within $R^2$ space: P (finite points set), L (finite lines set) and R (region). Their dimensions are defined as: $\text{dimension}(P) = 0$, $\text{dimension}(L) = 1$, $\text{dimension}(R) = 2$.

**Theorem 2** Given two objects $x, y$, if $\text{dimension}(x) < \text{dimension}(y)$ then $x$ can’t contain $y$.

Proof. This is true since infinite points exist within a line or a region and the infinite lines exist within a region.

Apparently all the basic relations of RCC5 [2] are possible in a multi-dimensional circumstance except the cases in Theorem 2. We call them MRCC5 (Table 1). The second spatial relation is qualitative size (QS), its basic relations are $\{<_S, =_S, >_S\}$ (Table 2). The last spatial relation is the qualitative diameter (QD). The diameter of an object is the maximum distance between two points on its boundary. Its basic relations are $\{<_D, =_D, >_D\}$.

#### Table 1: MRCC5 topological relations

<table>
<thead>
<tr>
<th></th>
<th>P</th>
<th>L</th>
<th>R</th>
</tr>
</thead>
<tbody>
<tr>
<td>P</td>
<td>PP-DR, PP-PO, PP-PP, PP-Ppi, PP-EQ</td>
<td>PL-DR, PL-PO, PL-PP</td>
<td>PR-DR, PR-PO, PR-PP</td>
</tr>
<tr>
<td>L</td>
<td>LP-DR, LP-PO, LP-Ppi</td>
<td>LL-DR, LL-PO, LL-PP, LL-Ppi, LL-EQ</td>
<td>LR-DR, LR-PO, LR-PP</td>
</tr>
<tr>
<td>R</td>
<td>RP-DR, RP-PO, RP-Ppi</td>
<td>RL-DR, RL-PO, RL-Ppi</td>
<td>RR-DR, RR-PO, RR-PP, RR-Ppi, RR-EQ</td>
</tr>
</tbody>
</table>

Function Dependency is true for all the items listed in Table 3; otherwise it’s false. For PLRTSD, 14 items out of 45 are true. Here * means the disjunction of all the basic relations within a relational model.
Table 2: Qualitative size relations

<table>
<thead>
<tr>
<th></th>
<th>P</th>
<th>L</th>
<th>R</th>
</tr>
</thead>
<tbody>
<tr>
<td>P</td>
<td>Compare the numbers of two point sets</td>
<td>( &lt;_S )</td>
<td>( &lt;_S )</td>
</tr>
<tr>
<td>L</td>
<td>( &gt;_S )</td>
<td>Compare the total length of two line sets</td>
<td>( &lt;_S )</td>
</tr>
<tr>
<td>P</td>
<td>( &gt;_S )</td>
<td>( &gt;_S )</td>
<td>Compare the area of two regions</td>
</tr>
</tbody>
</table>

Table 3: The dependency for the basic relations of QS, QD and RCC5

<table>
<thead>
<tr>
<th>RCC5</th>
<th>Size</th>
<th>Diameter</th>
</tr>
</thead>
<tbody>
<tr>
<td>DR</td>
<td>*</td>
<td>*</td>
</tr>
<tr>
<td>PO</td>
<td>*</td>
<td>*</td>
</tr>
<tr>
<td>PP</td>
<td>( &lt;_S )</td>
<td>( &lt;_D )</td>
</tr>
<tr>
<td>PP</td>
<td>( &lt;_S )</td>
<td>( =_D )</td>
</tr>
<tr>
<td>Ppi</td>
<td>( &gt;_S )</td>
<td>( =_D )</td>
</tr>
<tr>
<td>Ppi</td>
<td>( &gt;_S )</td>
<td>( &gt;_D )</td>
</tr>
<tr>
<td>EQ</td>
<td>( =_S )</td>
<td>( =_D )</td>
</tr>
</tbody>
</table>

Theorem 3: V is a variable set on multi-domains \{P,L,R\}. \( \Theta \) is a basic MRCC5 constraints set. \( \nabla \) is a basic QS constraints set. \( \Omega \) is a basic QD constraints set. MMCSP(V, \( \Theta \cup \nabla \cup \Omega \)) is consistent iff MMCSP(V, \( \Theta \cup \nabla \cup \Omega \)) is multi-algebraically-closed.

Proof: \( \Rightarrow \): MMCSP(V, \( \Theta \cup \nabla \cup \Omega \)) is globally consistent then it is also multi-algebraically-closed (locally consistent). \( \Leftarrow \): If MMCSP(V, \( \Theta \cup \nabla \cup \Omega \)) is multi-algebraically-closed, we can build a model of V in \( R^2 \) space by the following steps:

1. Build a model satisfying MRCC5.

We revise Li’s canonical realization method on RCC5 [9]. The block consists of three parts: region \( B^2 \) is a square, line \( B^1 \) is the diagonal of the square and points \( B^0 \) are two end points of the line (See Fig. 1(a) ). With the following steps we can build the model of MRCC5 constraints \( \Theta \) when it is algebraically closed.

(a) Merge all the variables with \{XX-EQ\}. (X stands for \{P,L,R\})
(b) For every variables \( x \), create a new block B, and label \( B^{dimension(x)} \) with ‘\( x \)’.  
(c) For \( x \{XX-PO\}y \ (i < j) \), create a new block B, label \( B^{dimension(x)} \) with ‘\( x \)’, label \( B^{dimension(y)} \) with ‘\( y \)’.
(d) For \( x \{XX-PP\}y \) or \( y \{XX-Ppi\}x \), if ‘\( x \)’ exists in the label of B, then add ‘\( y \)’ to the label of \( B^{dimension(y)} \).

Therefore the model satisfies the MRCC5 constraints \( \Theta \) if it is algebraically closed.
(2) Modify the model to satisfy the Qualitative Size constraint $\nabla$ (Fig. 1(b)). Adjust the size of each block $B^2$ for region objects. Stretch or shrink $B^1$ within the diagonal of the block for line objects. Change the number of points in $B^0$ for point objects by adding a few points within the diagonal of the block.

(3) Adjust the diameter to fit Qualitative Diameter constraint $\Omega$. Suppose that $M$ is the minimum rectangle containing all the blocks of (1) and (2), the diameter (length of diagonal) of $M$ is $d$. By extruding the diagonal, we add a slim bar to each block of single object $x$ (Fig. 1(b)). Each bar must not touch other blocks. This can be arranged by putting all the blocks in a line. The height of the slim bar is far greater than $d$, so we can let the model satisfy $\Omega$ by adjusting the height of the slim bar. In the meantime, the width of the slim bar is too small to affect the QS constraints.

Finally, we build a model satisfying $\Theta, \nabla$ and $\Omega$.

5 Using PLRTSD in A Complex Spatial Query

PLRTSD can be applied to construct and optimize complex spatial queries in GIS. There are two steps in the optimization.

(1) Preprocessing of query by checking the consistency

If the constraints set in a query is inconsistent, the query definitely has no result. Then GIS operation is not necessary. By checking the consistency with the Multi-algebraically-closed algorithm, a lot of unnecessary GIS operations are saved.

(2) Reduce Query Conditions by Minimal Representation

Not all constraints are indispensable, because some can be deduced from others by weak composition operations. The minimal representation has the minimal number of constraints and all other constraints in the full set can be deduced. Apparently, applying minimal support set will greatly save the query execution time.

**Algorithm:** Minimal-Representation ($V$, $\bigcup_{1 \leq k \leq p} \nabla_k$)

Input: $MMCSP(V, \bigcup_{1 \leq k \leq p} \nabla_k)$

Output: Minimal representation of $MMCSP$
1. For every \((x_i, x_j, R) \in \text{MMCSP}\)

2. If \((x_i, x_j, R)\) equals the weak composition of other two items in MMCSP, then delete it from MMCSP.

The above optimization strategies were tested with simulation data. Queries were generated by 50 students. Each query includes 4 to 16 variables. Among all the 500 queries only 47 queries are consistent. It means about 90% query can be filtered in the preprocessing stage. Applying the minimal representation saves about 70% of the query execution time.

6 Conclusion

This paper deals with two new qualitative spatial reasoning integration problems, integrating more than two spatial relations and integrating multi-domains. These problems haven’t been investigated before. However, they are all practical problems in GIS and other applications. Both general frameworks and a concrete model are presented. Complex spatial queries can be constructed and optimized by our method. The simulation experiment demonstrates that our approach is useful for practical problems.

References


