Medical Image Registration Using a Real Coded Genetic Algorithm

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Abstract

In this paper, a real-coded genetic algorithm called DMLX-NUM is proposed to solve the problem of parameter optimization in medical image registration. It derived from LX-NUM algorithm. The composition of the algorithm is simple, and the method has no special requirements for generating the initial population. By this algorithm the ratio of registration success can be at 98%. The algorithm we are going to present in this paper is compared with other methods. Simulation indicated that the proposed algorithm displays excellent accuracy and robustness in MRI image registration.

Keywords: Image Registration; Genetic Algorithm; Non-uniform Mutation

1 Introduction

Image registration is an important problem and a fundamental task in image processing technique. Medical images provide information about pathology and related anatomy of the human body.

Generally, image registration methods can be loosely divided into two classes, which are time domain approach and frequency domain approach. The first approach has intensity-based method and landmark-based method etc.. The second approach include FFT-based [2][3] and Wavelet-based [3] method. Intensity-based methods are based on intensity similarity measures between two images, the measure of similarity between images are computed directly from image intensities. Mutual information (MI) is a generally applicable similarity measure [4], for image registration based on MI, registration of two images can be formulated as a search or an optimization problem, this leads to a three-dimensional optimization problem with many local extremal. In this paper, we focus on the parameter optimization of medical image registration.

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Various traditional optimization methods have been used for optimization problem of registration based on MI, for example Gauss-Newton optimization methods [5] and Powell [6]. The local minima is core of optimization algorithm, in this respect, heuristics algorithm is better than traditional optimization techniques, and it does not require to calculate derivatives of index function. In recent years, an increasing number of heuristic optimization algorithms have been used in medical image registration, such as simulated annealing (SA) [7], particle swarm optimization (PSO) [8] and Genetic algorithm [9].

GA, which was developed by Holland, has been widely used in engineering applications. It is well known, the initial population from which the solution starts to evolve affect the performance of GA. Rachid et al. [10] proposed CGA to solve this problem and avoid the risk of having too much individuals in the same region, the CGA used the notion of “ball” to define a “neighborhood” for each selected individual. To overcome the weakness of local search ability of GA, several “hybrid” methods have been proposed [11][12]. To improve the fine local tuning capabilities of a GA, which is a must for high-precision problems, the Non-uniform mutation (NUM) operator was introduced by Michalewicz and Janikow. Such a mutation causes global search of the search space at the beginning of the iterative process, but only an increasingly local exploitation later on [13]. NUM was used in conjunction with Laplace Crossover (LX) by KUSUM et al. in 2007 [14]. By using those algorithms, better results can be obtained for many optimization problems. However, for some problem the above algorithm fails to find the global solution (see section 3.2). Therefore, we propose here a dynamically mutation probability function and derived a so-called DMLX-NUM algorithm from LX-NUM. The DMLX-NUM is further applied in medical image registration. Simulations showed that the DMLX-NUM is suitable for registration, and can achieve very high accuracy and excellent robustness.

The paper is organized as follows. In Section 2, the problem description and modeling are introduced. In Section 3, literature review and analysis on LX-NUM method are described. In Section 4, the proposed optimization algorithm for registration is displayed in detail. The results of simulation experiments are discussed in Section 5 and some words of conclusion make up the Section 6.

2 Problem Description and Modeling

Fig. 1 is the image registration framework. MI is a popular similarity metric for intensity-based registration, the MI is used as the Metric in our paper. For two images A and B, mutual information $MI(A, B)$ computed from [15]

$$
MI(A, B) = \sum P_{A,B}(a, b) \frac{P_{A,B}(a, b)}{P_A(a) \cdot P_B(b)} 
= H(A) + H(B) - H(A, B).
$$

The interpretation of this equation form is that measures the distance between the joint distribution of image pixel values $P_{A,B}(a, b)$, and the joint distribution in case of independence of the image $P_A(a)$ and $P_B(b)$. $H(A)$ and $H(B)$ respectively are entropies of A and B, and $H(A, B)$ is the joint entropy with A and B. $H(A)$, $H(B)$ and $H(A,B)$ can be estimated from

$$
H(A) = - \sum_{i=1}^{La} P_i(a) log P_i(a),
$$
Fig. 1: Image registration framework

\[ H(B) = -\sum_{i=1}^{Lb} P_i(b) \log P_i(b), \]  
\[ \text{and} \]
\[ H(A, B) = -\sum_{i=1}^{La} \sum_{i=1}^{Lb} P_{ij}(a, b) \log P_{ij}(a, b). \]

In registration process, optimization is used to search the transformation parameters when the measure \( MI \) is maximized. Next we will discuss about a genetic algorithm application for medical image registration.

3 Literature Review and Analysis on LX-NUM Method

Kusum Deep et al. proposed a real coded crossover called Laplace Crossover (LX), LX was used in conjunction with Non-Uniform Mutation to define a new genetic algorithms LX-NUM.

In this section, we will to do a review on the LX-NUM method and analyze its application in medical image registration.

3.1 Laplace crossover (LX)

LX is parent centric operator. Using LX, two offsprings \( y^{(1)} = (y_1^{(1)}, y_2^{(1)}, \ldots, y_n^{(1)}) \) and \( y^{(2)} = (y_1^{(2)}, y_2^{(2)}, \ldots, y_n^{(2)}) \) are generated from a pair of parents \( x^{(1)} = (x_1^{(1)}, x_2^{(1)}, \ldots, x_n^{(1)}) \) and \( x^{(2)} = (x_1^{(2)}, x_2^{(2)}, \ldots, x_n^{(2)}) \) in the following way.

First, two uniformly distributed random numbers \( u_i, u_i' \in [0, 1] \) are generated. Then, a random number \( \beta_i \) is generated which follows the Laplace distribution by simply inverting the distribution function of Laplace distribution as follows:

\[ \beta_i = \begin{cases} 
  a - \log_e(u), & u_i \leq \frac{1}{2}, \\
  a + \log_e(u_i), & u_i > \frac{1}{2}, 
\end{cases} \]  
\[ \text{(5)} \]

The offsprings are given by the equations

\[ y_i^{(1)} = x_i^{(1)} + \beta \mid x_i^{(1)} - x_i^{(2)} \mid, \]  
\[ y_i^{(2)} = x_i^{(2)} + \beta \mid x_i^{(1)} - x_i^{(2)} \mid. \]  
\[ \text{(6, 7)} \]
3.2 Non-uniform mutation (NUM)

If individual $x = x_1, x_2, \cdots, x_n$ is a chromosome and the element $x'_k$ is selected for mutation (the domain of $x_k$ is $[U_{min}^k, U_{max}^k]$), the mutation result is a vector $x = x_1, x_2, \cdots, x', \cdots, x_n$

$$x'_k = \begin{cases} x_k + \Delta(t, U_{max}^k - x_k) & \text{if } r \leq 0.5, \\ x_k - \Delta(t, x_k - U_{min}^k) & \text{otherwise} \end{cases}$$ (8)

where $r$ is a uniformly distributed random number between 0 and 1. The function $\Delta(t, y)$ give below takes value in the interval $[0, y]$

$$\Delta(t, y) = y(1 - u(1 - \frac{t}{T}))^b$$ (9)

where $u$ is a uniformly distributed random number in the interval $[0,1]$, $T$ is the maximum number of generations and $b$ is a parameter.

We use the LX-NUM algorithm which is proposed by Kusum to optimize parameter of MRI image registration, and conduct simulation based on the reference [14]. Simulation results showed that, the mostly of registration results are failure. Because optimization performance is greatly effected by the parameter, we adjusted parameters, the success rate is higher when mutation probability equals 0.05, but there still are some failure cases. Fig. 2 is a case of failure in registration. From Fig. 2 we can see that, for registration problem, the LX-NUM algorithm hardly jump out from local minima when the average and optimal value difference is small, this causes the failure occurred in limited evolution generation.

Through the analysis of formula (8) and formula (9), we found that with the increase of the number of iterations, the mutation discrepancy value $\Delta x_k = |x'_k - x_k|$ becomes smaller and smaller, the results of mutation are almost same as the $x_k$ when the number of iteration reaches near to the max generation $T$. due to this reason, Non-uniform mutation have a high local search ability, but it also reduces the mutation ability. In addition, from the formulas we can see that, the speed reduction of $\Delta x_k$ also relates with $T$. If $T$ is defined very lager, then the speed reduction of $\Delta x_k$ becomes lower, for limited iterations some larger $\Delta x_k$ can be produced. Otherwise if the definition of Max generation $T$ is not very large, then the speed reduction of $\Delta x_k$ becomes quite fast. This leads to that, if the algorithm can’t jump from the local minima
in the primary iteration process, then the algorithm would be difficult to be outstretched near to
the global optimum when the iterations $\Delta x_k$ become smaller and smaller. Therefore, we propose
a dynamic mutation probability formula to update the mutation probability with the number of
generation. Thus, although $\Delta x_k$ reduces gradually in the evolution progress, algorithm increases
probability to jump out of local minima and retains the merit which is stronger local search
ability of Non-uniform due to the increase of mutation probability and the number of mutation
individuals.

4 Proposed Optimization Algorithm

In this section, we describe the details of the DMLX-NUM algorithm: LX-NUM with a Dynamic
updating Mutation probability.

4.1 Gene and chromosome formulation

Transformation of image can be represented by three parameters $(t_x, t_y, \theta)$, a set of parameters
is defined as a chromosome. Each parameter then corresponds to one of the genes in the chro-
mosome. The translation and rotation parameters of image form a chromosome $[t_x, t_y, \theta]$ which
represents the relation between two images.

\[
\begin{bmatrix}
  x' \\
  y' \\
  1
\end{bmatrix} = \begin{bmatrix}
  k \cos \theta & -k \sin \theta & t_x \\
  k \sin \theta & k \cos \theta & t_y \\
  0 & 0 & 1
\end{bmatrix} \begin{bmatrix}
  x \\
  y \\
  1
\end{bmatrix}
\]  

(10)

will represent the components of transformation function that register the images. The mutual
information is as Fitness function. For convenience, $(x_1, x_2, x_3)$ is represented as a vector of
chromosome to the solution of the $(t_x, t_y, \theta)$. The initialization of population is generated by

\[
x(i) = r \times (U_{i_{\text{max}}} - U_{i_{\text{min}}}) + U_{i_{\text{min}}}
\]  

(11)

where $r$ is a uniformly distributed random number in the interval $[0,1]$, $U_{\min}$ and $U_{\max}$ is the lower
and upper bound of $t_x$, $t_y$ and $\theta$.

4.2 Detailed description of our algorithm

Tournament selection is used as the selection operator in our work. Randomly select $m$ individuals
from the population to take part in the tournament (the $m$ is tournament size), in our paper $m$ is
define as 2. Choose the individual that has the highest fitness value from the individuals selected
above by comparing the fitness value of each individual. Then the chosen one is copied into the
next generation of the population. The process is repeated $n$ times, where $n$ is the number of
individuals [16].

The Laplace crossover is used as crossover operator, and crossover index $b$ is define as 0.15.
Here, we proposed a dynamic mutation probability $P_m$, the $P_m$ is defined by

\[
P_m = 1 - e^{-\gamma t/T}
\]  

(12)

where $t$ is current number of generation, $T$ is max generation, $\gamma$ is seted as 0.4 in our work.
Fig. 3: Test images: (a) Original MRI image; (b) Moving image; (c) Corrupted by Poisson noise; (d) Moving image.

**Pseudo-code:**

DMLX-NUM

1: to generate a set of initial chromosomes at random

Repeat

2: the worst individual is replaced with global best individual

   IF global best fitness < best fitness of the population THEN

3: global best individual = best individual of the population

4: global best fitness = best fitness of the population

   END

5: Selection (apply Tournament selection)

6: Crossing (apply Laplace crossover)

7: dynamic updating the mutation probability \( P_m \)

8: Mutation (apply Non-uniform Mutation operator)

9: fitness calculation

Until MaxGen

Here, the mutation index \( b \) of Non-uniform Mutation is defined as 4.

5 Experiment Results and Discussion

The 8-bit 256 × 256 MRI images of human head used for the experiments are shown in Fig. 3. Because medical images are often noisy owing to the physical mechanisms of the acquisition process, and the Poisson noise is suitable for modelling the counting processes associated to many imaging modalities [17]. Among test images, two images are noise free image, two images are corrupted by poisson noise. Fig. 3 (a) is the original image, (b) is the image was translated 6 pixel in the x direction and was translated 8 pixel in the y direction rotated 15 degree from the original image. (c) is the image (a) corrupted by Poisson noise, (d) is image (c) was translated 6 pixel in the x direction and was translated 8 pixel on the y direction rotated 15 degree.

The four images are divided into two groups. The first group include (a) and (b). (c) and (d) are taken as second group. We see the registration of two groups images as two optimization
problems. The results of simulations of these optimization algorithms were compared, the three algorithms are LX-NUM, LX-MPTM and proposed algorithm, respectively.

We have run each algorithm 100 times in both problems. To evaluate the algorithm efficiency, the number of successful runs, average number of function evaluations and average execution time of successful runs listed in Table 1 and Table 2. For our experiments, a run is said to be a successful run if the best mutation information (MI) error is within 0 and 0.0001. From Tab. 1 and Tab. 2, it is observed that, for standard MRI image registration, the proposed algorithm outperforms LX-NUM and LX-MPTM in terms of average number of function evaluations and average time of execution as well as the number of successful runs. For second group with noise images registration problem, the average number of evaluations using proposed algorithm is bigger than LX-NUM, but the ratio of success is quite high, it can achieved 98% success rate. Furthermore, the time of execution is smaller than another algorithm.

### Table 1: No. of successful runs, average no. of function evaluations and execution time of successful runs for LX-NUM and proposed algorithm

<table>
<thead>
<tr>
<th>Problem number</th>
<th>Number of successful runs (out of 100 runs)</th>
<th>Average number of function evaluations of successful runs</th>
<th>Average time of execution in s</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>LX-NUM</td>
<td>PROPOSED</td>
<td>LX-NUM</td>
</tr>
<tr>
<td>1</td>
<td>86</td>
<td>95</td>
<td>1012.2</td>
</tr>
<tr>
<td>2</td>
<td>79</td>
<td>98</td>
<td>884.8</td>
</tr>
</tbody>
</table>

### Table 2: No. of successful runs, average no. of function evaluations and execution time of successful runs for LX-MPTM and proposed algorithm

<table>
<thead>
<tr>
<th>Problem number</th>
<th>Number of successful runs (out of 100 runs)</th>
<th>Average number of function evaluations of successful runs</th>
<th>Average time of execution in s</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>LX-MPTM</td>
<td>PROPOSED</td>
<td>LX-MPTM</td>
</tr>
<tr>
<td>1</td>
<td>82</td>
<td>95</td>
<td>1198.2</td>
</tr>
<tr>
<td>2</td>
<td>78</td>
<td>98</td>
<td>1141.0</td>
</tr>
</tbody>
</table>

In order to get a better insight into the relative performance of LX-NUM, LX-MPTM and proposed algorithm, the value of a performance index ($PI$) [18] is calculated in respect of these three algorithms. This index gives a weighted importance to the success rate, the computational time as well as the number of function evaluations. The value of this performance index for a computational algorithm under comparison is given by

$$PI = \frac{1}{Np} \sum_{i=1}^{Np} \left( k_1 \frac{S^i_r}{T^i_r} + k_2 \frac{M^i_t}{A^i_t} + k_3 \frac{M^i_f}{A^i_f} \right).$$

(13)

where, $Np$ is the total number of test problems which have been solved by each of the algorithms under comparison, $S^i_r$ is number of successful runs of $i$th problem, $T^i_r$ is total number of runs of $i$th problem, $A^i_t$ is the average time used by all the algorithms in obtaining the solution of $i$th problem, $Mt^i_t$ is the minimum of average time used by all the algorithms in obtaining the solution of $i$th problem, $A^i_f$ is the average number of function evaluations required by the algorithm in
obtaining the optimal solution of the $i$th problem in the case of successful trials (its value is assumed to be zero if all the trials are unsuccessful), and $Mf^i$ is the minimum of the average number of function evaluations required by the various algorithms under comparison in solving the $i$th problem. From the above definition it is clear that the $PI$ is a function of $k_1, k_2$ and $k_3$ ($0 \leq k_1, k_2, k_3; k_1 + k_2 + k_3 = 1$). Same as ref. [14], the $k_1, k_2$ and $k_3$ are assigned as follows:

(i) $k_1 = w, k_2 = k_3 = \frac{1-w}{2}, 0 \leq w \leq 1$,
(ii) $k_2 = w, k_1 = k_3 = \frac{1-w}{2}, 0 \leq w \leq 1$,
(iii) $k_3 = w, k_1 = k_2 = \frac{1-w}{2}, 0 \leq w \leq 1$,

The graphs of $PI$ corresponding to each of these three cases are shown in Fig. 4 – 6. In Fig. 4, weights is assigned to the percentage of success as $k_1 = w$, and to average time of successful run ($k_2$) and average function evaluation of successful run ($k_3$) as $k_2 = k_3 = (1 - w)/2$. Similarly, in Fig. 5, weight assigned to the average time of successful run ($k_2$) is $k_2 = w$, for number of success $k_1$ and average function evaluation of successful run ($k_3$) are $k_1 = k_3 = (1 - w)/2$. The Fig. 4 and Fig. 5 illustrate that the $PI$ value of the proposed algorithm is better than other algorithms. In Fig. 6, for weights $k_3 = w, k_1 = k_2 = (1 - w)/2$, the performance index of proposed algorithm is better than another two algorithms when $w < 0.65$. But in the latter part of the $w$, the $PI$ value of proposed algorithm is lower than LX-NUM algorithm, this occurs because the number of function evaluations is considered to be important than to the success rate and execution time about $w > 0.65$. Since in most practical applications, we pay attention to the success rate and execution time more than to the number of function evaluations, we may consider that the proposed algorithm results are satisfactory for image registration.

The maximum numbers of generations are fixed to be 50 for all of the GAs in simulation, and crossover probability is define as 0.5, population size is 50. All the algorithms are implemented in Matlab and the experiments are done on CPU 2.6GHZ machine with 512 MB RAM under WINXP platform.
6 Conclusion

In this paper, we analyze the reasons of low success rate of LX-NUM in MRI image registration. A dynamic updating mutation probability is proposed. The proposed algorithm is applied to MRI image registration. Registration accuracy, PI and other aspects are compared. Comparisons indicate that the proposed so-called DMLX-NUM algorithm can be applied effectively to MRI image registration. From the optimization of view, the proposed algorithm can find a global optimal solution in comparatively less execution time and number of evaluation function. The experiments on registration of noise MRI images is performed, and the robustness of the proposed algorithm was verified.
References