An Optimization Algorithm of Cooperative Spectrum Sensing in Cognitive Radio Networks

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Abstract

By taking into account the reporting channel fading and the differences in the reliability of the decisions made by different secondary users (SUs), a general performance analysis of cooperative spectrum sensing are given. The analysis results show that the performance of cooperative spectrum sensing is limited by the probability of reporting error and decision fusion rule. To deal with the limitation, an optimal algorithm cooperative spectrum sensing is proposed. Firstly, a transmit diversity with randomized space-time coding is proposed for local sensing decisions. Next, an optimal weighted fusion rule with the likelihood ratio test is proposed, and the optimal detection threshold of energy detection is determined numerically. The simulation results show that the proposed optimal algorithm has improved the performance of spectrum sensing.

Keywords: Cognitive Radio Network; Cooperative Spectrum Sensing; Fusion Rule; Detection Threshold; Random Space-time Coding

1 Introduction

To improve the performance of spectrum sensing, cooperation amongst the SUs has been proposed [1,2,3,4], the key idea of which is that spectrum detection decision is based on those of multiple distributed SUs instead of a local SU alone.

Typically, cooperative spectrum sensing requires two successive stages, sensing and reporting. In practice, since the reporting channels between the SUs and the central unit also suffer some interferences or noise, they do affect the efficiency of spectrum sensing. With the notable exception of [5] no existing literature fully incorporates them into discussion. In this letter, we consider cooperative spectrum sensing in a realistic environment where both the sensing channels and reporting channels are characterized by fading channels. In [5], a transmit diversity based cooperative spectrum sensing algorithm which applies Alamouti space-time coding is proposed.

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However, only a simple case of two SUs was considered. In addition, cooperative fusion scheme affect the sensing performance. In [6,7], k-out-of-n rule has been proposed and all the SUs are assumed to have the same detection SNRs. However, in particular, SUs don’t have the same detection SNRs in most cases. For the cooperative spectrum sensing with energy detection, the detection threshold affect detection performance, as a consequence, how to optimize the detection threshold is an important issue for cooperative spectrum sensing.

The main contributions of this paper are described as follows. The minimization of the total error probability in terms of the various parameters is investigated. Firstly, we proposed a transmit diversity based on randomized space-time coding to transmit local sensing decisions. It decreases the probability error rate of reporting channel and works for any number of SUs. Secondly, in this paper, an efficient decision fusion rule which take into account the differences in the reliability of the decisions made by different SUs is proposed. Thirdly, we also determine the optimal detection threshold to minimize the error rate.

2 System Model

We consider a CR network composed of SUs and a secondary base station (SBS) which manage all the SUs, as shown in Fig. 1. During each cooperative spectrum sensing process, there are two essential phases: 1) local detection phase, where all SUs attempt to detect the presence of the primary user (PU); 2) reporting phase, where each SU transmit its initial detection result to the SBS such that the SBS can make a final decision on the presence of PU by using a decision fusion rule.

![Cognitive radio network](image)

Fig. 1: Cognitive radio network

The local sensing process is modeled by using energy detector [7], the probabilities of detection and false alarm at the SUi are, respectively as [8]

\[
p_{f,i} = \frac{\Gamma \left( u, \frac{\lambda_i}{2} \right)}{\Gamma(u)}
\]

\[
p_{d,i} = e^{-\frac{\lambda_i}{2}} \sum_{n=0}^{u-2} \frac{1}{n!} \left( \frac{\lambda_i}{2} \right)^n + \left( \frac{1+\gamma_i}{\gamma_i} \right)^{u-1} \times \left[ e^{-\frac{\lambda_i}{2(1+\gamma_i)}} - e^{-\frac{\lambda_i}{2}} \sum_{n=0}^{u-2} \frac{1}{n!} \left( \frac{\lambda_i \gamma_i}{2(1+\gamma_i)} \right)^n \right]
\]

Where \( \Gamma(.) \) is gamma function, \( \lambda_i \) and \( f_s \) denote the energy threshold and the sampling frequency, the instantaneous signal-to-noise ratio (SNR) at the SUi is thus given by \( \gamma_i \).
**Definition 1** $p_{e,i}$, the probability of reporting errors of the $SUi$, which denotes the error probability of signal transmission over the reporting channel between the $SUi$ and the SBS.

**Definition 2** $P_{f,i}$ and $P_{d,i}$ are the false alarm probability and the detection probability of $SUi$ at the SBS. Suppose that the probabilities of reporting errors $p_{e,i}$ are identical for all SUs, $p_{e,i} = p_e (\forall i)$, then,

$$P_{f,i} = (1 - p_{f,i})p_e + p_{f,i}(1 - p_e)$$  \hspace{1cm} (3)

$$P_{d,i} = (1 - p_{d,i})p_e + p_{d,i}(1 - p_e)$$  \hspace{1cm} (4)

**Definition 3** $x = [x_1, x_2, ..., x_n]$, it is the received decisions at the SBS, there will be $2^n$ possible outcomes of $x$.

**Definition 4** $D^+ = \{x | \omega = 1\}$, it is the set of values that the SBS will decide that the PU is present.

The probability of the overall probability of detection, the overall probability of missed detection and the overall probability of false alarm, respectively shown as

$$Q_d = \sum_{x \in D^+} P(x|H_1)$$  \hspace{1cm} (5)

$$Q_m = 1 - Q_d$$  \hspace{1cm} (6)

$$Q_f = \sum_{x \in D^+} P(x|H_0)$$  \hspace{1cm} (7)

Assume independence of the SU’s decisions, $P(x|H_1)$ and $P(x|H_0)$ are, respectively, expressed as

$$P(x|H_1) = \prod_{i=1}^{n} P_{d,i}^{x_i} (1 - P_{d,i})^{1 - x_i}$$  \hspace{1cm} (8)

$$P(x|H_0) = \prod_{i=1}^{n} P_{f,i}^{x_i} (1 - P_{f,i})^{1 - x_i}$$  \hspace{1cm} (9)

### 3 Proposed Optimization Scheme of Cooperative Spectrum Sensing

It has been shown that the performance of cooperative spectrum sensing is limited by the probability of reporting error $p_{e,i}$, decision fusion rule $D^+$ and the detection threshold $\lambda_i$. In the proposed algorithm, we investigate the minimization of the total error probability in terms of the various parameters. The proposed algorithm flowchart as follows
3.1 Transmit diversity with randomized space-time coding

In order to decrease the effect of imperfect reporting channel, in this section, we will employ transmit diversity with randomized space-time coding to improve the performance of cooperative spectrum sensing by reducing \( p_e \).

The system between SUs and the SBS can be seen as a MISO system with \( n \) transmits and 1 receive antenna. A Rayleigh fading channel is assumed, where the channel gains are constant during each fading interval and independent in successive intervals. It can be shown that the channel vector is, respectively, given by

\[
H = \{h_1, h_2, ... h_n\}^T
\]  

(10)

Where \( h_i \) is the fading channel coefficient between SU\( i \) and the SBS. The received signal \( Y \) at the SBS has the following form

\[
Y = \sqrt{\frac{\bar{\gamma}}{n}} TH + N
\]  

(11)

Where \( \bar{\gamma} \) is the average received SNR of SUs, \( T = [t_1, t_2, ..., t_n] \) is the randomization space-time coding matrix, \( H \sim N_c(0, \Sigma H)N \sim N_c(0, N_0I) \).

Let \( d = \{d_1, d_2, ..., d_n\} \) be the vector of local decisions, we assume that \( d \) is perfectly to the SUs that participate in the cooperation. At each of SU, \( d \) is mapped onto a matrix \( G \) as is done in standard space-time coding. Let \( R = \{r_i\}_{i=1}^n \), \( r_i \) is the random vector that contains the linear combination coefficients for the SU\( i \), then, \( T \) can be expressed as

\[
T = G \ast R
\]  

(12)

Let \( G \) employ Alamouti scheme and the randomization is done in uniform phase randomization, hence

\[
G = \begin{bmatrix}
  s_1 & s_2 \\
  s_2^* & -s_1^*
\end{bmatrix}
\]  

(13)

Where \([s_1, s_2]\) is the transmitted symbol vector.

\[
R = \begin{pmatrix}
  e^{j\theta_{11}} & e^{j\theta_{12}} & \cdots & e^{j\theta_{1n}} \\
  e^{j\theta_{21}} & e^{j\theta_{22}} & \cdots & e^{j\theta_{2n}}
\end{pmatrix}
\]  

(14)

Where \( \theta_{ij} \) is a random variable uniformly distributed in \([0, 2\pi]\).
Definition 5 $|A|_{k+} = \prod_{i=1}^{k} \delta_{m-i+1}$, it is the product of $k$ smallest positive eigenvalues of the matrix $A$. $A$ is an $n \times n$ matrix with $\delta_1 \geq \delta_2 \geq \ldots \geq \delta_m > 0 \geq \delta_{m+1} \geq \ldots \geq \delta_n$.

Let $SNR = 1/N_0$, if the matrix $R$ is full rank with probability 1 and the expectation $E\{|RR^H|_{2+}^{-1}\}$ is infinite, then the $P_e$ is bounded as [9]

$$P_e \leq \alpha SNR^{-2} E\{|RR^H|_{2+}^{-1}\}$$

(15)

Where $\alpha = \frac{1}{16 \min(i,j)(\|G_i - G_j\|^H \|G_i - G_j\|\sum_k \|x_k\|_2^2)}$.

The number of SUs do not limited a certain value because of the randomization of $R$, so, the proposed method is more general.

3.2 Optimal fusion rule

In this section, an optimal fusion rule has been considered for a more general case that the SUs are assumed to have different detection SNRs.

For an effective fusion rule, the main metric of sensing performance is minimizing the total error $P_E$. It is shown as follows

$$P_E = P(H_0)P(H_1|H_0) + P(H_1)P(H_0|H_1) = \pi_0 P(H_1|H_0) + \pi_1 P(H_0|H_1)$$

(16)

Theorem 1 Suppose that $n$ and SNRs are known, the optimal fusion rule is denoted as $\tilde{D}^+$,

$$\tilde{D}^+ = \{x = (x_1, \ldots, x_n) | \sum_{i=1}^{n} [(1 - x_i)\alpha_i + x_i\beta_i] \geq \log \frac{\pi_0}{\pi_1} \}$$

(17)

Where $\alpha_i = \log \frac{1 - P_{d_i}}{P_{f_i}}$, $\beta_i = \log \frac{P_{f_i}}{P_{d_i}}$ are weighting factors for the SU’s decisions that do not detect the PU and the SU’s decisions that detect the PU.

Proof Define $R_1 = \{x|H_1\}$ is the decision domain that PU present. $R_0$ is its complementary set. From (16), we get

$$P_E = P(H_0) \int_{R_1} P(x|H_0)dx + P(H_1) \int_{R_0} P(x|H_1)dx$$

(18)

The entire space is divided by $R_0$ and $R_1$, so

$$\int_{R_0} P(x|H_1)dx = 1 - \int_{R_1} P(x|H_1)dx$$

(19)

From (19), (18) can be further expressed as

$$P_E = \int_{R_1} P(H_0) P(x|H_0) - P(H_1) P(x|H_1)dx + P(H_1)$$

(20)
If the integral is negative, $x$ is included in, or
\[ P(H_0)P(x|H_0) \leq P(H_1)P(x|H_1) \] (21)
x is included in $R_1$, therefore,
\[ \frac{P(x|H_1)}{P(x|H_0)} \geq \frac{P(H_0)}{P(H_1)} = \frac{\pi_0}{\pi_1} \] (22)

From (8) and (9), we can get
\[ \frac{P(x|H_1)}{P(x|H_0)} = \prod_{i=1}^{n} \frac{P_{d,i}^{x_i}(1 - P_{d,i})^{1-x_i}}{P_{f,i}^{x_i}(1 - P_{f,i})^{1-x_i}} \] (23)

And by taking logarithm on both sides, from (22), we can get
\[ LLR(X) = \sum_{i=1}^{n} \left[ (1-x_i)\alpha_i + x_i\beta_i \right] \geq \log \frac{\pi_0}{\pi_1} \] (24)
Where $\alpha_i = \log \frac{1-P_{d,i}}{1-P_{f,i}}$, $\beta_i = \log \frac{P_{d,i}}{P_{f,i}}$.

### 3.3 Optimal energy threshold

The performance of local sensing is limited to the energy threshold. However, for a big CR network, it may not be possible and practical for all the SU to computing their optimal threshold. This is because it will bring huge complexity and cost. In this paper, we assume that all the SUs have a same energy threshold.

Suppose that $n$, $D^+$ and SNRs are known, the optimal energy threshold which minimizes the total error probability is $\bar{\lambda}$,
\[ \bar{\lambda} = \arg \min_{\lambda} [Q_f + Q_m] \] (25)

It is the solution of the equation $\frac{\partial Q_f}{\partial \lambda} + \frac{\partial Q_m}{\partial \lambda} = 0$. From (1), (7) and (9), we can get
\[ \frac{\partial Q_f}{\partial \lambda} = \frac{\partial \sum_{x \in D^+} \prod_{i=1}^{n} P_{f,i}^{x_i}(1 - P_{f,i})^{1-x_i}}{\partial \lambda} \] (26)
from (5), (6) and (8), we can get
\[ \frac{\partial Q_m}{\partial \lambda} = \frac{\partial \left[ 1 - \sum_{x \in D^+} \prod_{i=1}^{n} P_{d,i}^{x_i}(1 - P_{d,i})^{1-x_i} \right]}{\partial \lambda} \] (27)

Using (26) and (27), the solution can be evaluated numerically. It is the optimal energy threshold.
4 Performance Evaluation

In this section, we evaluate the performance of our proposed algorithms through simulations. The CR network is composed with 4 SUs, one PU and the SBS. Sensing channel and reporting channel is both Raleigh channel. The sensing channels have different SNRs, and reporting channels have average SNR 13 dB. \( \pi_0 = \pi_1 = 1/2 \), and the simulation result is averaged over 1000 experiments.

Fig. 3 shows the ROCs for different decision fusion rule, the averaged instantaneous SNRs of SUs are random within the range between -10dB-5dB. Randomized space-time coding is used for local decision transmit. As the simulation shows, the proposed fusion rule has the best performance. The superiority is more obvious at low SNR.

Fig. 4 shows the total probability of error detection for various fusion rules. Optimal energy threshold and randomized space-time coding are employed. For a given SNR value \( \delta \) dB in the figure, the average instantaneous SNR of each SU is uniformly selected from \( ((\delta - 2) - (\delta + 2)) \) dB. It can be observed from the figure, the optimal fusion rule has the least total error rate.

Fig. 5 shows the total probability of error detection for TDMA and transmit diversity technique. Optimal energy threshold and decision fusion rule have been employed. For a given SNR value \( \delta \) dB in the figure, the average instantaneous SNR of each SU is uniformly selected from \( ((\delta - 2) - (\delta + 2)) \) dB. It can be observed from the figure, the performance of proposed algorithm has a good performance. This is because the randomized space-time coding increases the diversity order and coding gain.

Fig. 6 illustrates the total probability of error detection versus the SNR for the proposed and
traditional algorithms. The TDMA scheme and And-Rule is used for traditional algorithms, the energy threshold is not the optimal value. As shown in Fig. 6, compared with the traditional algorithm, the proposed scheme has a smaller total error probability. It is a good proof that the proposed algorithm can improve the detection performance.

5 Conclusion

In this paper, Cooperative spectrum sensing over Rayleigh fading channels in CR network with SU having different SNRs was studied. In the proposed algorithm, a transmit diversity with randomized space-time coding was applied for cooperative sensing to reduce the probability of reporting errors. Meanwhile, it has been found that the optimal decision fusion rule to minimize the total error probability is a weighted rule based on likelihood ratio test. Finally, the optimal energy threshold is determined numerically in the proposed algorithm. The algorithms greatly improve the performance of cooperative spectrum sensing.

References


