An Adaptive Fuzzy Variable Structure Controller for Bank-to-Turn Missile

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Abstract
This paper address the control problem for the bank-to-turn (BTT) missile, which is a complicated nonlinear system. BTT autopilot is not simple, mainly due to the nonlinear and coupled character of system equations, the lack of a precise model for the missile dynamics and parameters, as well as the appearance of internal and external perturbations. The proposed methodology is an approach included in the control areas of nonlinear feedback linearization, fuzzy control and uncertainties consideration. It is based on the fusion of a sliding mode controller and fuzzy basis function networks (FBFN), including the advantages of both systems. The main advantage of this methodology is that it relaxes the required knowledge of vehicle model. The experimental results demonstrate the good performance of the proposed controller, within the constraints of the sensorial system and the uncertainty of the theoretical models.

Keywords: Adaptive Control; Variable Structure Control; Bank-to-Turn (BTT) Missile; Autopilot

1. Introduction
Bank-to-turn (BTT) missiles have high maneuverability as compared with skid-to-turn (STT) missiles[1]. To execute a BTT autopilot strategy, these missiles must have the ability to change the orientation of the acceleration rapidly by means of a very high roll rate. However, such a high roll rate will induce cross coupling, resulting in undesirable pitch and yaw motion, so autopilot design for a bank-to-turn (BTT) tactical missile is inherently nonlinear, with highly coupled dynamics, as well as the appearance of internal and external perturbations[2]. These difficulties create a need for the use of multivariable and nonlinear control techniques[3].

To solve the problem, input/output (I/O) feedback linearization was introduced to obtain the plant dynamics in the autopilot design of BTT missile[4-5]. However, it is difficult to establish or identify an accurate dynamic model of a complicated BTT missile system for designing optimal controller. One of reducing the plant dynamics is using the neural networks (NN), which can be a universe approximator to the unknown nonlinear functions and un-modeled errors. Based on this method[6-7], stable adaptive updating parameter laws are derived by the Lyapunov theory to guarantee tracking performance and closed-loop stability. Also, the robust control methods have emerged to to minimize the level of approximation errors and external disturbances, such as in [8-9].

Fuzzy logic control law can be designed based on some knowledge or without any knowledge about the control system[10-11]. In addition, an appropriate fuzzy logic controller can overcome the environmental variation during flight processes. Therefore, it has been employed in the field of BTT missile[12-13]. According to [10], the fuzzy system can be represented as series expansions of fuzzy basis functions to form a network, which is called fuzzy basis function network (FBFN), as like as the RBF neural networks[14]. FBFN has the capability of combining both numerical data and linguistic information, which

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has good ability of function approximation[14]. Hence, FBFN can be employed to the BTT autopilot to approximate the plant dynamics, while variable structure control is an attractive method for controlling BTT missiles to handle the approximation errors and external disturbances[15-16].

Based on the aforementioned analysis, we propose a new approach to design the BTT autopilot, which is based on the fusion of a sliding mode controller and FBFN. The main advantage of this methodology is that it relaxes the required knowledge of vehicle model.

2. Adaptive Fuzzy Variable Structure Controller

2.1. Fuzzy Basis Function Networks (FBFN)

Considering a fuzzy system with the following form of rules:

\[ R^j: \text{IF } x_1 \text{ is } A^j_1 \text{ and } x_2 \text{ is } A^j_2 \text{ and } \cdots \text{ and } x_n \text{ is } A^j_n, \text{ THEN } y \text{ is } B^j \]

where \( R^j \) is the \( j \)th rule, \( j=1,2,3,\ldots,M \) is the number of fuzzy rules, \( x=(x_1,x_2,\ldots,x_n)\in U \subset R^n \) denotes the output variable of fuzzy system and \( i=1,2,3,\ldots,n \) is the number of input variable, \( y \in W \subset R \) represents the output variable, \( A^j_i \) and \( B^j \) are linguistic terms in the discourse \( U \) and \( W \).

The fuzzy system, with singleton fuzzification, product inference, and defining the defuzzifier as a weighted sum of each rule's output, can be described as follows

\[ y(x) = \sum_{j=1}^{M} \xi_j(x) \theta_j = \xi^T(x) \theta = \theta^T \xi(x) \]

where \( \theta=(\theta_1,\theta_2,\ldots,\theta_M)^T \) is the tunable-parameter vector, \( \theta_j \) is the maximum value according to the \( B^j \), \( \xi=(\xi_1,\xi_2,\ldots,\xi_M)^T \) are the fuzzy basis functions (FBF).

The fuzzy basis function is defined as[10]:

\[ \xi_j(x) = \frac{\prod_{i=1}^{n} \mu_{A^j_i}(x_i)}{\sum_{j=1}^{M} \prod_{i=1}^{n} \mu_{A^j_i}(x_i)} \quad (j=1,2,\ldots,M) \]

where \( \mu_{A^j_i}(x_i) \) is the Gaussian membership function.

Fig.1 shows the structure of FBFN, which is like the RBF networks, the comparison of them can be found in [14].

From (3), it is easy to find out that \( \sum_{j=1}^{M} \xi_j(x) = 1 \) and \( 0 \leq \xi_j(x) \leq 1 \).
According to the document [10], the FBFN can approximate any function in the continuous course with arbitrary accuracy, i.e.:

$$\sup_{x \in \Omega} |f(x) - g(x)| \leq \varepsilon$$

(4)

2.2. Controller Design

The variable structure control theory was proposed by Utkin [17] in 1977. The stability problem and application of variable structure systems had been developed in reference [15]. It has been widely employed to control nonlinear dynamic systems[18-20]; especially the systems have model uncertainties and external disturbances. Here, the sliding mode concept is combined with fuzzy control strategy to design a model-free adaptive fuzzy sliding mode controller for nonlinear systems control.

A class of nonlinear multiple-input-multiple-output nth-order system can be written in the following form:

$$y^{(n)} = f(y^{(n-1)}, \ldots, y, y, t) + \Delta f(y^{(n-1)}, \ldots, y, y, t) + g(y^{(n-1)}, \ldots, y, y, t)u + d(t)$$

(5)

where $y \in \mathbb{R}^n$ is the output vector, $f(y^{(n-1)}, \ldots, y, y, t) \in \mathbb{R}^m$ and $g(y^{(n-1)}, \ldots, y, y, t) \in \mathbb{R}^{m \times m}$ are the nonlinear functions of the system, and $u \in \mathbb{R}^m$ represents control input, $\Delta f(y^{(n-1)}, \ldots, y, y, t) \in \mathbb{R}^m$ being the uncertainties and $d(t) \in \mathbb{R}^m$ being the external disturbances.

Define $x_1 = y, x_2 = \dot{y}, \ldots, x_n = y^{(n-1)}$, and the system (5) can be rewritten in such way:

$$\begin{align*}
\dot{x}_1 &= x_2 \\
\dot{x}_2 &= x_3 \\
&\vdots \\
\dot{x}_n &= f(X, t) + \Delta f(X, t) + g(X, t)u + d(t)
\end{align*}$$

(6)

where the state vector is $X = [x_1^T, x_2^T, \ldots, x_n^T]^T = [x_1^T, x_2^T, \ldots, x_{(n-1)}^T]^T$.

Assumption: The uncertainties $\Delta f(X, t)$ and the external disturbances $d(t)$ meet the following conditions

$$|f(X, t)| \leq F(X, t), \quad |d(t)| \leq D(t)$$

(7)

Both $F(X, t)$ and $D(t)$ are non-negative functions.

The control objective is to find a control law for the input torque $u$ which ensure the tracking errors $E$ converge to zero in a limited time where

$$E = X_d - X = [e^T, \dot{e}^T, \ldots, e^{(n-1)}]^T$$

(8)

$$e = x_{1d} - x_1 = [e_1, e_2, \ldots, e_m]^T$$

(9)

$$X_d = [x_{1d}^T, x_{2d}^T, \ldots, x_{nd}^T]^T = [x_{1d}^T, x_{1d}^T, \ldots, x_{(n-1)d}^T]^T$$

is the desired state vector.

Applying equation (8) and (9) to the system (6), then the system can be further rewritten as

$$\begin{align*}
\dot{e}_1 &= e_2 \\
\dot{e}_2 &= e_3 \\
&\vdots \\
\dot{e}_n &= x_d^{(n-1)} - f(X, t) - \Delta f(X, t) - g(X, t)u - d(t)
\end{align*}$$

(10)

Since it is difficult even impossible to measure all the state variables in practical implementation, here the state variables are selected to define a sliding surface on the phase plane.
\[ s(X,t) = -CE \]  
(11)

where \( C = [C_1, C_2, \ldots, C_n] \), \( C_i = \text{diag}(c_{i1}, c_{i2}, \ldots, c_{in}) \), \( c_{ij} > 0 \) for \( i = 1,2,\ldots,n; j = 1,2,\ldots,m \), which are selected by the designer.

If all the nonlinear terms in the previous equation are well known, a perfect control law with feedback linearization can be defined as

\[ u_{eq} = g^{-1}(X,t) \left( x_{id}^{(n)} + \sum_{k=1}^{n-1} C_k e^{(k)} - f(X,t) - \Delta f(X,t) - d(t) - u_{sw} \right) \]  
(12)

where \( u_{sw} = \eta \text{sgn}(s), \eta > 0 \).

\[ \dot{s} = -u_{sw} = -\eta \text{sgn}(s) \]  
(13)

\[ s\dot{s} = -\eta |s| \leq 0 \]  
(14)

With this perfect control law, the closed loop control system has an asymptotical stability dynamic behavior. Based on the definition of sliding surface variable \( s \) in (11), the selected system-states output errors \( E \) will gradually converge to zero, too.

Actually, it is difficult to know \( f(X,t), \Delta f(X,t), g(X,t) \) and \( d(t) \) in the practical system, then the perfect control law cannot obtain. To solve this problem, the fuzzy systems are employed to approximate the functions \( f(X,t), \Delta f(X,t), g(X,t) \) and \( d(t) \), then equation (12) can be changed into

\[ u_{eq} = u_{FBF} = \hat{g}^{-1}(X,t) \left( x_{id}^{(n)} + \sum_{k=1}^{n-1} C_k e^{(k)} - \hat{f}(X,t) - \hat{h}(s,t) \right) \]  
(15)

where \( \hat{h}(s,t) = \Delta f(X,t) - \eta \text{sgn}(s) \) represents the system’s uncertainties and external disturbances.

According to the fuzzy approximation theory, then \( \hat{f}(X \mid \theta_f), \hat{g}(X \mid \theta_g) \) and \( \hat{h}(s \mid \theta_h) \) can be expressed as in the FBFN

\[ \hat{f}(X \mid \theta_f) = \theta_f^T \xi_f(X) \]  
(16)

\[ \hat{g}(X \mid \theta_g) = \theta_g^T \xi_g(X) \]  
(17)

\[ \hat{h}(s \mid \theta_h) = \theta_h^T \xi_h(s) \]  
(18)

where \( \xi_f(X) \), \( \xi_g(X) \), \( \xi_h(s) \) are FBFs, and \( \theta_f, \theta_g \) and \( \theta_h \) are the parametric variables

\[ \hat{h}(s \mid \theta_h^*) = \eta_h \text{sgn}(s) \]  
(19)

where \( \eta_h = \eta + D \).

Our purpose is to construct a fuzzy control \( u_{FBF} \) by developing adaptive laws for the parameter vectors, where

\[ \dot{\theta}_f = -\gamma_f s \xi_f(X) \]  
(20)

\[ \dot{\theta}_g = -\gamma_g s \xi_g(X)u \]  
(21)

\[ \dot{\theta}_h = -\gamma_h s \xi_h(s) \]  
(22)

where \( \gamma_f, \gamma_g, \gamma_h \) are positive numbers.

2.3. Stability Analysis
Lyapunov stability analysis is the most popular approach to prove and evaluate the convergence property of nonlinear controllers, e.g., sliding mode control, fuzzy control system. Here, Lyapunov analysis is employed to investigate the stability property of the proposed controller.

**Theorem:** Consider the nonlinear plant (6) and control law (15) with the adaptive laws (20), (21) and (22), then, there exists a Lyapunov function

\[
V = \frac{1}{2} \left( s^2 + \frac{1}{\gamma_f} \phi_f^T \phi_f + \frac{1}{\gamma_g} \phi_g^T \phi_g + \frac{1}{\gamma_h} \phi_h^T \phi_h \right)
\]

satisfying \( \dot{V} < 0 \).

where

\[
\phi_f = \theta_f^* - \theta_f
\]
\[
\phi_g = \theta_g^* - \theta_g
\]
\[
\phi_h = \theta_h^* - \theta_h
\]

and

\[
\theta_f^* = \arg \min_{\theta_f \in \Omega_f} \left[ \sup_{t \in \mathbb{L}} \left| f(X | \theta_f) - f(X, t) \right| \right]
\]
\[
\theta_g^* = \arg \min_{\theta_g \in \Omega_g} \left[ \sup_{t \in \mathbb{L}} \left| g(X | \theta_g) - g(X, t) \right| \right]
\]
\[
\theta_h^* = \arg \min_{\theta_h \in \Omega_h} \left[ \sup_{t \in \mathbb{L}} \left| h(s | \theta_h) - h(s, t) \right| \right]
\]

\( \Omega_f, \Omega_g \) and \( \Omega_h \) are the sets of \( \theta_f, \theta_g \) and \( \theta_h \), respectively.

**Proof:** Defining the least approximation-error as the following form

\[
w = \hat{f}(X | \theta_f) - f(X, t) + \left( \hat{g}(X | \theta_g) - g(X) \right) \theta_h^* \)
\]

and

\[
\dot{s} = CE = C_e [ \hat{e}_f, \hat{e}_g, \ldots, \hat{e}^{(n)}_f ]^T = C_e e^{(n)} + \sum_{k=1}^{n-1} C_k e^{(k)} = C_n \left( \hat{x}_e - x^{(n)}_e \right) + \sum_{k=1}^{n-1} C_k e^{(k)}
\]

where we can choose \( C_n = \text{diag}(1, 1, \ldots, 1) \) for convenient computation, equation (31) should be further written as

\[
\dot{s} = \hat{f}(X, t) + \Delta f(X, t) + g(X, t) u + d(t) - x^{(n)}_e + \sum_{k=1}^{n-1} C_k e^{(k)}
\]

And the derivative of the Lyapunov function

\[
\dot{V} = \frac{1}{2} \left( \dot{s}^2 + \frac{1}{\gamma_f} \dot{\phi}_f^T \dot{\phi}_f + \frac{1}{\gamma_g} \dot{\phi}_g^T \dot{\phi}_g + \frac{1}{\gamma_h} \dot{\phi}_h^T \dot{\phi}_h \right)
\]
\[
\begin{align*}
\dot{V} &= s\dot{s} + \frac{1}{\gamma_f} \varphi_f \dot{\varphi}_f + \frac{1}{\gamma_g} \varphi_g \dot{\varphi}_g + \frac{1}{\gamma_h} \varphi_h \dot{\varphi}_h \\
&\leq \frac{1}{\gamma_f} \varphi_f \left( \gamma_f s \xi_f(X) + \dot{\varphi}_f \right) + \frac{1}{\gamma_g} \varphi_g \left( \gamma_g s \xi_g(X)u + \dot{\varphi}_g \right) + \frac{1}{\gamma_h} \varphi_h \left( \gamma_h s \xi_h(s) + \dot{\varphi}_h \right) \\
&\quad + sw - \eta |s| \\
&\leq sw - \eta |s|
\end{align*}
\]

According to the fuzzy approximation theory, fuzzy system can approximate any continuous function within any arbitrary accuracy, and (33)

\[
\dot{V} \leq 0
\]

3. Simulation

According to the 6-DOF dynamic equations of BTT missiles given in Appendix A, the nominal plant of BTT missiles with mismatching uncertainties can be described by

\[
y^{(n)} = f(X, t) + \Delta f(X, t) + g(X, t)u + d(t)
\]

where \( X = [\alpha, \beta, P, Q, R, U, V, W, \Phi, \Theta, \Psi]^T, \ u = [u_p, u_q, u_r]^T, \ y = [P, \alpha, \beta]^T, \ \Delta f(X, t) \) denotes the uncertainty due to the I/O feedback linearization, \( d(t) \) being the external disturbances and the elements of matrix \( g(X, t) \) are like in [13].

Consider the nonlinear plant (35) and control law (15) with the adaptive laws (20), (21)and (22). The parameters are chose like the following.

\[
\begin{align*}
g &= 9.807 \text{m/s}^2, \quad c &= 295.0464 \text{m/s}, \quad m &= 144.2951 \text{kg}, \quad k_f = 0.01425 \text{m}^2, \\
k_g &= 2.716 \times 10^{-3} \text{m}^3, \quad \rho_{aw} = 0.26 \text{kg/m}^3, \quad I_{xx} = 1615 \text{kg/m}^2, \quad I_{yy} = 136.24 \text{kg/m}^2, \\
I_{zz} &= 136.33 \text{kg/m}^2, \quad C_1 = \text{diag}(4, 4, 4), \quad C_2 = \text{diag}(4, 4, 4), \quad C_3 = \text{diag}(1, 1, 1), \quad \gamma_f = 2, \\
\gamma_g &= 1, \quad \gamma_h = 6 \quad \text{The fuzzy membership functions are NB, NM, ZO, PM, PB.}
\end{align*}
\]

Fig.2(a) and Fig.2(b) are the simulation results, which show that the system can track the desired commands well.

4. Conclusion

This paper presents a novel adaptive fuzzy sliding mode controller is proposed and successfully employed to control the BTT missile. The main advantage of this methodology is that it relaxes the required knowledge of vehicle model. The experimental results demonstrate the good performance of the proposed controller, within the constraints of the sensorial system and the uncertainty of the theoretical models.

Reference

Fig. 2(a) Simulation Results

Fig. 2(b) Simulation Results


