An Improved NMR Signal De-noising Algorithm Based on Wavelet Transform

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Abstract

The amplitude of nuclear magnetic resonance (NMR) longing signal general is very small while the signal to noise ratio (SNR) is also very low, so de-noising NMR signal before T2 spectrum inversion is necessary and important. In this paper, an improved de-noising algorithm based on wavelet transform is put forward. The main idea of this improved algorithm is that NMR signal is divided to several sub-window signals according to the exponential decay character of free induction decay (FID) signal and different sub-window signal is de-noised by wavelet transform using an improved threshold function. The detailed steps of this improved algorithm are discussed in this paper based on the steps of wavelet-based threshold de-noising method. In order to verify the performance of this improved algorithm, a 32-point small porosity model is designed and a free induction decay signal is generated by this model. Four algorithms are applied to filter the NMR noisy signal in an experiment: median filtering algorithm, wavelet transform filtering algorithm, FIR filtering algorithm and the improved algorithm, SNR and RMSE of these algorithms are also calculated in this paper. The result of this experiment displays that the improved de-noising algorithm based on wavelet transform, comparing to other algorithms, can improve the SNR of NMR signal and more effective keep useful signal.

Keywords: Nuclear Magnetic Resonance; Wavelet Transform; De-noising; Threshold; Signal to Noise Ratio; Free Induction Decay Signal

1. Introduction

Nuclear Magnetic Resonance (NMR) logging is a direct and effective technology for oil exploration, which has been widely applied to petroleum well logging and rock core analysis since the 1990s when NUMAR introduced a reliable NMR logging tool to the oil industry [1]. As we know that the hydrogen nuclear has the feature of spin and different pore fluid has different relaxation characteristic, so the free induction decay (FID) signal that produced by nuclear magnetic resonance logging can be detected. The porosity, T2 spectrum and other geological evaluation parameters, which are very important to oil and gas exploration, are calculated through NMR signal [2, 3]. However, the amplitude of NMR signal is small and the signal to noise ratio (SNR) of NMR signal is also low, one of the most important works before T2 spectrum inversion is how to enhance NMR signal [4]. Filtering NMR signal is necessary, which not only increases the SNR of NMR signal, but also improves the stability and continuity of T2 spectrum inversion.

There are many algorithms to enhance the signal. Signal averaging is used to increase SNR, but this often results in a huge increase in NMR machine time, which may not be acceptable in many situations
Conventional Fourier transform techniques is a good tool, but it is not good for NMR signal because NMR signal and noise overlap in the spectrum. Wavelet transform is an effective time-frequency domain method for noise reduction, which is used to filter NMR signal, but many researches focused on nuclear magnetic resonance images. The paper [5] used wavelet transform to de-noise the images of NMR signal, and put forward the choice method about parameter selection of wavelet transform. Wang et al pay attention to that the wavelet transform had been used to do-noise the medical nuclear magnetic resonance free induction signal in reference [6] and three de-noised method based on wavelet transform had been also discussed in this paper, but the method of this paper only suit to the medical nuclear magnetic resonance. Xie et al discussed a stein unbiased risk estimation (SURE) algorithm based on wavelet transform in reference [7], which is used to de-noise the NMR logging signal. In paper [8] a new de-noising method of NMR logging signals based on empirical mode decomposition had been discussed, which was more powerful to increase SNR comparing with some points smoothing method.

In this paper, we put forward a new improved de-noising algorithm of NMR signal based on wavelet transform. The main idea of this improved algorithm is that we divide NMR signal into many sub-window signals according to the characteristic of NMR signal, any sub-window signal is one part of the NMR signal. Different sub-window signal can be de-noised by different wavelet transform and an adaptive threshold function, this paper also discusses the adaptive threshold function. This improved algorithm can improve signal to noise ratio (SNR) and maintain the smoothness of de-noised signal, which also can improve the quality of the T2 spectrum inversion.

This paper is organized as follows. In section 2 we will review wavelet transform as well as the steps of wavelet-based threshold de-noising method. In section 3 we will introduce the steps of our improved wavelet transform algorithm, followed by the experimental evaluations and analysis in section 4. The conclusions are given in section 5.

2. Wavelet-based Threshold De-noising Method Review

Wavelet transform is an effective time-frequency (TF) domain method for signal analysis, which has good localization properties in time domain as well as in frequency domain. It is widely used in image compression, edge detection, noise reduction and so on. Continuous wavelet transform is defined as:

$$W_s(a,b) = \left( s(t), \psi_{a,b}(t) \right) = \frac{1}{\sqrt{a}} \int_{-\infty}^{+\infty} s(t) \psi^* \left( \frac{t-b}{a} \right) dt$$

$$\psi_{a,b}(t) = \frac{1}{\sqrt{a}} \psi \left( \frac{t-b}{a} \right)$$

(1)

Where $\psi_{a,b}(t)$ is the mother wavelet function while $a$ and $b$ represent the time shift factor and the scale expansion factor respectively. Wavelet transform can be understood as the inner product at different scales between the signal $s(t)$ and the basic wavelet function (mother wavelet function) which has been displaced [9].

There are three methods for wavelet transform to reduce noise: Modulus Maxima Method, Scale Space Filtering Method and Domain Value Method [10, 11]. Modulus maxima method and scale space filtering method require the modulus maxima of signal increasing and the noise decreasing with the size of signal.
growing. In contrast, the wavelet-based threshold de-noising method is more effective, which uses wavelet transform to decompose the signal $s(k)$ in the frequency domain and de-noises the coefficients at different scales. The block diagram of wavelet-based threshold de-noising method is shown in Figure 1.

$$\begin{align*}
\text{Step 1.} & \quad \text{Wavelet transform of the noisy signal.} \\
& \quad \text{First, select an orthogonal wavelet basis and determine the scale coefficients } j. \text{ Then wavelet transform of the noisy signal with orthogonal wavelet basis, the approximation coefficients and the detail coefficients at all scale are calculated.} \\
\text{Step 2.} & \quad \text{Process the resulting wavelet coefficients at each scale with threshold value.} \\
& \quad \text{Second, choose different domain value to threshold the resulting wavelet coefficients at each scale.} \\
\text{Step 3.} & \quad \text{At last, inverse of wavelet transform to obtain the de-noised signal.} \\
& \quad \text{Wavelet-based threshold de-noising method is an effective tool to de-noise, however, NMR signal is full}
\end{align*}$$

Fig.1 Wavelet-based Threshold De-noising Method

In Fig 1, $w^i_\phi(i, k)$ and $w^i_\psi(i, k)$ are called the detail and the approximation coefficients of the signal $s(k)$ at scale $i$ respectively, where $i = 1, \ldots, l$. The threshold function $d'_{j,k}$ is used to de-noise all detail coefficients and the approximation at scale $j$, while $\hat{s}(k)$ represents the de-noised signal after signal reconstruction with IDWT (Inverse Discrete Wavelet Transform). There are three stages in this wavelet-based threshold de-noising method.

**Step 1.** Wavelet transform of the noisy signal.

First, select an orthogonal wavelet basis and determine the scale coefficients $j$. And then wavelet transform of the noisy signal with orthogonal wavelet basis, the approximation coefficients and the detail coefficients at all scale are calculated.

**Step 2.** Process the resulting wavelet coefficients at each scale with threshold value.

Second, choose different domain value to threshold the resulting wavelet coefficients at each scale.

There are two threshold rules, hard-threshold and soft-threshold, are often used. $d'_{j,k}$ is supposed to denote the detail coefficient. The general hard-threshold function is defined as

$$d'_{j,k} = \begin{cases} d_{j,k}, & |d_{j,k}| > \lambda_j \\ 0, & \text{otherwise} \end{cases}$$

(2)

While soft-threshold function is defined as

$$d'_{j,k} = \begin{cases} \text{sgn}(d_{j,k}) \|d_{j,k}\| - \lambda_j, & |d_{j,k}| > \lambda_j \\ 0, & \text{otherwise} \end{cases}$$

(3)

Where $\lambda_j$ represents the threshold at scale $j$, and it is very important to affect the quality of de-noising algorithm. Many improved wavelet transform algorithm are concerned with that how to optimize this threshold function or $\lambda_j$.

**Step 3.** At last, inverse of wavelet transform to obtain the de-noised signal.

Wavelet-based threshold de-noising method is an effective tool to de-noise, however, NMR signal is full
of noise and SNR is very low. So an improved de-noising algorithm based on threshold de-noising method has been put forward in this paper in accordance with the characteristics of NMR signal.

3. Improved De-noising Algorithm

Any discrete NMR signal can be well modeled as a finite mixture of modulated exponential functions plus noise [3, 12]. The following formula 4 can be used to explain this discrete NMR signal.

\[
s(k) = y(k) + \varepsilon(k) = \sum_{i=1}^{n} f_i * e^{-r(k)/T_{2i}} + \varepsilon(k) \tag{4}
\]

Where \( \varepsilon(k) \) is the noise, \( k \) is the length of NMR signal, \( n \) is called the number of relaxation spectrum distribution while \( T_{2i} \) is supposed to the pre-specified series of relaxation time distribution. The existence of noise in NMR signal may lead to the instability or discontinuity of T2 spectrum in inverse algorithm, so NMR signal is required to de-noise before T2 spectrum inverse in order to improve SNR and the quality of inverse algorithm. The flow chart of this improved algorithm is described in figure 2.

![Flow Chart of Improved De-noising Algorithm](image)

In Fig 2, \( s(k) \) is the NMR origin signal which includes the noisy signal while \( k \) is the length of NMR signal, \( s'(i) \) represents the de-noised signal, \( s_j(i,a,b) \) is one part of NMR signal, which is called sub-window signal, \( a \) and \( b \) mean the start subscript and the end subscript of the NMR origin signal respectively. The steps of this improved de-noising algorithm based on wavelet transform are explained as follow.

1. **Wavelet Transform**
2. **Average Filtering**
3. If \( \frac{p_{l}}{p_{s}} > 0.1 \)
   - **Yes**
   - **Wavelet Transform**
   - **No**
   - **Average Filtering**
4. If \( j = n \)
   - **Yes**
   - **combination**
5. **No**
   - \( j = j + 1 \)

Fig.2 Flow Chart of Improved De-noising Algorithm
Step 1. Cut the first and the second data of NMR origin signal.

General, the first and the second data of NMR signal is inaccurate, which has been polluted by noise and be full of randomness. So the first and the second data of NMR signal are cut before de-noising in order to keep the useful signal.

Step 2. Add windows to NMR signal

NMR signal is divided to several sub-window signals according to the following formula 5 [13]. Every sub-window signal is one part of NMR signal with different length.

\[ NW_i = N_i^{(i-1)}; N_i = NW_i - NW_{i-1} \]  \hspace{1cm} (5)

Where \( NW_i \) is the sampling function while \( N \) is the length of NMR origin signal, \( n \) is the number of sub-windows and \( i = 1, 2, \ldots, n \), it is mean that NMR signal is separated into \( n \) sub-window signals while \( N_i \) represents the length of sub-window signal \( s_i \).

NMR signal is called NMR free induction decay signal (FID) that is because NMR signal displays exponential decay, furthermore, the amplitude of useful signal is much large than noise’s at the front half of NMR signal, but it is full of noise at the latter. So different sub-window signals have different SNR through adding windows to NMR signal. It is worth notice that the number of windows general is 15 to 32 as well as associated with the length of NMR origin signal.

Step 3. Wavelet transform of the sub-window signal.

Get the detain coefficients and the approximation coefficients of the sub-window signal through wavelet transform.

Step 4. Threshold processing

The hard threshold function and the soft threshold function have been discussed above in this paper. There is an improved threshold function, which is defined as formula 6:

\[
\hat{d}_{j,k} = \begin{cases} 
  k \cdot \text{sgn}(d_{j,k}) \cdot \left|d_{j,k}\right| - \lambda_j, & \text{if } \left|d_{j,k}\right| > \lambda_j \\
  0, & \text{otherwise}
\end{cases}
\]  \hspace{1cm} (6)

Where \( k = 0.5 \) in this improved threshold processing, the scale-dependent threshold at scale \( j \) have been presented in the reference [14]:

\[ \lambda_j = \sigma_j \sqrt{2 \ln N} \]  \hspace{1cm} (7)

Where \( N \) is the length of sub-window signal and \( \sigma_j \) is the variance distribution of noise at scale \( j \). The adaptive threshold is defined as:

\[ \lambda_{j,k} = \frac{p_k}{p_s} \sigma_j \sqrt{2 \ln N} \]  \hspace{1cm} (8)

Where \( p_k \) is the power of \( k \) sub-window signal while \( p_s \) is the sum power of NMR origin signal. If \( p_k / p_s < 0.1 \), then the sub-window signal is de-noised by average filtering algorithm.

Step 5. Obtain the de-noised sub-window signal by inverse discrete wavelet transform.
4. Experiments

In order to verify performance of this improved de-noising algorithm, the NMR signal has been simulated. In this experiment the median filter algorithm, wavelet transform filtering algorithm, FIR filter algorithm and this improved algorithm are used to de-noise the simulated NMR signal.

4.1. Experimental Setup

First, the model of small porosity has been constructed by using 32-point T2 relaxation spectrum according to experiences and the data form rock analyzer as been shown in fig 3. The minimum time of axis x is 0.5ms while the maximum is 2048ms. The porosity is displayed by axis y and the max value is 2.08 pu. Figure 4 shows the weak NMR signal with adjustable SNR that is generated by the model of small porosity. The value of axis y is relative in fig 4.

4.2. Experimental Results

In this experiment, the improved de-noising algorithm, comparing to median filter algorithm, wavelet transform algorithm and FIR filter algorithm are used to process the NMR signal. The wavelet transform algorithm selects "db4" as wavelet basis and decomposes NMR signal to 3 layers using the hard threshold function; The FIR filter algorithm uses 4800points to sample and the cutoff frequency is designed 100Hz. This improved de-noising algorithm decomposes NMR signal to 32 sub-window signals, selects different wavelet basis for different sub-window signals and uses adaptive improved threshold to de-noise. The de-noised signal of four filtering algorithm are shown in fig 5.
It is very clear that the de-noised signal of median filter algorithm and FIR algorithm is relatively poor, comparing with other de-noising algorithm. Much high frequency noises still stay in the de-noised signal of median filter algorithm and FIR algorithm, which are shown in fig 5 (1) and fig 5 (2). The effect of wavelet transform algorithm is better than the median filter algorithm and the FIR algorithm. It is obvious that there are few noises in the de-noised signal of the wavelet transform algorithm form the fig 5(3). However, the de-noise signal of the improved de-noising algorithm is the best, there is not only few noises still exist in this de-noised signal but also it is more smooth than the de-noise signal of the wavelet transform, which can be saw in the fig 5(4). One conclusion form fig 5 is that the effect of the improved de-noising algorithm is good to filter NMR signal comparing to other algorithms.

At the same time, there are two parameters to describe the performance of de-noising algorithm. One is SNR and the other one is root mean square error (RMSE), which are defined as formula 9 and 10.

\[ \text{SNR} = 10 \times \log_{10} \left( \frac{\sum_i |s_i|^2}{\sum_i |s_i - s'_i|^2} \right) \]  
(9)

\[ \text{RMSE} = \sqrt{\frac{1}{n} \sum_i (s_i - s'_i)^2} \]  
(10)

where \( n \) is the length of signal, \( s_i \) and \( s'_i (i = 1, 2, \cdots n) \) represent the noisy signal and the de-noised signal respectively.

Table 1 shows three group data about SNR and RMSE of four de-noising algorithm. What we can see form table 1 is that the SNR of the improved de-noising algorithm is the highest comparing the other algorithms at the same conditions. Form the first column of table 1, the SNR of this improved de-noising algorithm is 25.16, which is much larger than others. The same results can be saw form the third and the fifth column of table 1. The RMSE of this improved de-noising algorithm is always the minimum value,
which can be got form the second, the forth and the sixth column of table 1, comparing the other algorithms. The result of table 1 shows that the improved algorithm can make SNR better and enhance signal.

<table>
<thead>
<tr>
<th>Algorithm</th>
<th>SNR1(DB)</th>
<th>RMSE1</th>
<th>SNR2(DB)</th>
<th>RMSE2</th>
<th>SNR3(DB)</th>
<th>RMSE3</th>
</tr>
</thead>
<tbody>
<tr>
<td>Median filter</td>
<td>12.74</td>
<td>0.366</td>
<td>15.85</td>
<td>0.254</td>
<td>22.34</td>
<td>0.119</td>
</tr>
<tr>
<td>Wavelet transform</td>
<td>9.92</td>
<td>0.504</td>
<td>13.06</td>
<td>0.353</td>
<td>19.99</td>
<td>0.159</td>
</tr>
<tr>
<td>FIR</td>
<td>7.01</td>
<td>0.706</td>
<td>8.34</td>
<td>0.606</td>
<td>9.96</td>
<td>0.498</td>
</tr>
<tr>
<td>Improved algorithm</td>
<td>25.16</td>
<td>0.316</td>
<td>25.83</td>
<td>0.249</td>
<td>28.48</td>
<td>0.117</td>
</tr>
</tbody>
</table>

5. Conclusion

This paper reviewed wavelet transform as well as the steps of wavelet-based threshold de-noising method and proposed an improved de-noising algorithm based on wavelet transform for NMR signal. The improved threshold function was also discussed in this paper followed by the steps of the improved de-noising algorithm. The model of small porosity had been designed and the NMR signal was simulated in an experiment, this paper used the improved de-noising algorithm, comparing the median filter algorithm, wavelet transform filtering algorithm and FIR algorithm to filter the simulated NMR signal. There are two conclusions what we can get form this experiment through comparing the de-noised signal of this four algorithms and calculating the performance parameters (SNR, RMSE) of these four de-noising algorithms. The first conclusion is that the effect of the improved de-noising algorithm to filter NMR signal is better than others while the other conclusion is that the improved de-noising algorithm is good to improve SNR and keep the useful NMR signal.

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References


