Balanced Guides Assignment for Multi-Objective Particle Swarm Optimizer

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Abstract

Recently, more and more studies to apply particle swarm optimization (PSO) algorithm for solving multi-objective problems (MOPs) have been proposed. Due to there will be more than one global best solution gbest found can be solution searching clue in MOPs. For leading more particles toward to potential searching space and find global optimum, suitable local guide (global best solution) assignment becomes an important issue in multi-objective particle swarm optimizer. This paper presents a density-based local guide assign approach called balanced guide assignment (BGA) for multi-objective particle swarm optimization. Besides, for making proposed method more robust, a cluster archive approach and perturbation method are also involved with BGA. They can efficiently improve particles’ solution searching ability to find more solutions located on/near to the Pareto front. Six benchmarks were adopted for testing and compare the proposed method with other related works. From the results, the proposed method performed better in performance metrics can be observed.

Keywords: Balanced Guide Assignment (BGA); Local Guide; Multi-Objective Optimization; Pareto Front; Particle Swarm Optimization (PSO).

1. Introduction

The main different between single-objective optimization problems (SOPs) and multi-objective optimization problems (MOPs) is that MOPs contain more than one objective that needs to be achieved simultaneously. Such problems arise in many applications, where two or more, sometimes competing and/or incommensurable objective functions have to be minimized concurrently.

Since Schaffer proposed a Vector Evaluated Genetic Algorithm (VEGA) [1][2] in 1984, many multi-objective optimization (MO) algorithms have then been proposed. In [3], Fonseca and Fleming adopted GA to solve MO problems named multi-objective GA (MOGA). The found non-dominated solutions are classified and then ranked to enhance the searching abilities for finding non-dominated solutions, and to maintain the diversity of the found solutions. But it cannot ensure that a solution with a worse rank will always be mapped to a worse fitness value. This will give the algorithm slower convergence and more instability. Srinivas and Deb proposed a Non-dominated Sorting Genetic Algorithm (NSGA) [4], which ranks populations according to its characteristic of non-domination, and gives higher

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fitness values for better non-dominated solutions. The Niched Pareto Genetic Algorithm (NPGA) [5] was proposed by Horn et al. It introduced a binary tournament selection but does not assign a definite fitness value, but problems with more optimized objectives will influence the computational efficiency of NPGA.

After that, several efficient strategies have been introduced based on these algorithms, such as Elitism, archive, or archive. Zitzler and Thiele proposed the Strength Pareto Evolutionary Algorithm (SPEA) [6], which introduced an elitism strategy to store an extra population that contains non-dominated solutions. New found non-dominated solutions will be compared with the extra stored population, and the better is kept. The SPEA2 [7] is an advanced version of SPEA. SPEA2 inherited the advantages from SPEA and improved fitness assignment to take both dominated and non-dominated solutions into account. SPEA2 also considered neighboring solutions’ diversity to produce more capable guides. Similar to MOGA, all these approaches have the same problem; the non-dominated solutions with the same ranks may not have the same status. In [8], Knowles et al. proposed Pareto Archive Evolution Strategy (PAES), which employs (1+1) evolution strategy (ES) and uses a mutation operator for local searches. A map of a grid is applied in the algorithm to maintain the diversity of the archive. Thus, there will be a trade-off to define the size of both the external repository and grid of the map. Deb proposed an enhanced NSGA named NSGA-II [9][10] which employs a fast non-dominated approach to assign ranks to individuals and crowded tournament selection for density estimation. In the case of a tie in rank during the selection process, the individual with a lower density count will be chosen.

Besides, many researches are interesting in PSO solving multi-objective problems [11]-[14]. For transforming an original PSO (single-objective) into a multi-objective PSO (MOPSO), a guide must be redefined in order to obtain a set of non-dominated solutions (Pareto front). In MOPSO, the Pareto-optimal solutions should be used to determine the guide for each particle. The selection of suitable local guides for attaining both convergence and diversity of solutions becomes an important problem. Hu and Eberhart proposed a dynamic neighborhood PSO [13], which optimizes only one objective at a time and uses a scheme similar to lexicographic ordering. In [12], Fieldsend and Singh proposed an unconstraint elite archive named dominated tree to store the non-dominated solutions, but it’s a difficult issue for this approach to pick up a best local guide from the set of Pareto-optimal solutions for each particle of the population. Coello Coello et al. proposed a MOPSO method which incorporates Pareto dominance and a special mutation operator to solve multi-objective optimization problems [15]. A strategy for finding suitable local guides for each particle was proposed by Mostaghim and Teich named sigma method [16]. The local guide is explicitly assigned to specific particles according to the sigma value. This result in desired diversity and convergence but its still not close enough to the Pareto front.

On the other hand, an enhanced archiving technique to maintain the best (non-dominated) solutions found during the course of a MO algorithm was proposed in [17]. It shows that using archives in PSO for MO problems will improve their performance directly. Recently, Parsopoulos and Tasoulis proposed a vector evaluated particle swarm optimization (VEPSO) [18] which adopted a ring migration topology and PVE system to simultaneously work 2 to 10 CPUs to find non-dominated solutions. The MOPSO method is becoming more popular due to its simplicity to implement and its ability to quickly converge to a reasonably acceptable solution for problems in science and engineering. Recently, Ho and Tay [19] adopted evolution and local search to solving flexible job shop problems (FJSP).

To increase particles’ solution searching ability, in this paper, a local guide global best solution
assignment approach called balanced guide assignment is proposed for MOPSO. Through this strategy, a suitable global best solution will be assigned to each particle in the swarm according to current solution searching status. Thus, Particles will be more capable to find more solutions located on/near to the Pareto front. Further more, a cluster based archive approach and Perturbation method are also introduced to keep desired population diversity form the non-dominated solution set found so far.

The paper is organized as follows. The particle swarm optimization is introduced in Section II. The balanced guide assignment for MOPSO is described in Section III. The test functions, experimental settings and results are presented in Section IV. Section V of the paper contains the conclusions.

2. Particle Swarm Optimization

The PSO is a population based optimization technique that was proposed by Kennedy and Eberhart [20] in 1995, which the population is referred to as a swarm. The particles express the ability of fast convergence to local and/or global optimal positions over a small number of generations.

A swarm in PSO consists of a number of particles. Each particle represents a potential solution of the optimization task. All of the particles iteratively discover a probable solution. Each particle moves to a new position according to the new velocity and the previous positions of the cell. This is compared with the best position generated by previous particles in the cost function, and the best one is kept; so each particle accelerates in the direction of not only the local best solution but also the global best position. If a particle discovers a new probable solution, other particles will move closer to it to explore the region more completely in the process [21].

Let \( s \) denote the swarm size. In general, there are three attributes, current position \( x_i \), current velocity \( v_i \) and past best position \( p_{best_i} \), for particles in the search space to present their features. Each particle in the swarm is iteratively updated according to the aforementioned attributes. Assuming that the function \( f \) is to be minimized so that the dimension consists of \( n \) particles, the new velocity of every particle is updated by

\[
v_{i,j}(g+1) = wv_{i,j}(g) + c_1r_{i,j}(g)[p_{best,i,j}(g) - x_{i,j}(g)] + c_2r_{i,j}(g)[g_{best,j}(g) - x_{i,j}(g)]
\]

(1)

For all \( j \in \{1, \ldots, n\} \), \( v_{i,j} \) is the velocity of \( j^{th} \) dimension of the \( i^{th} \) particle the, \( c_1 \) and \( c_2 \) denote the acceleration coefficients, \( r_1 \) and \( r_2 \) are elements from two uniform random sequences in the range \((0, 1)\), \( g \) is the number of generations and of the \( w \) is called inertia weight of velocity into the original PSO, introduced by Shi and Eberhart [22]. The new position of a particle is calculated as follows

\[
x_{i}(g+1) = x_{i}(g) + v_{i}(g+1)
\]

(2)

The past best position (pbest) of each particle is updated using

\[
p_{best,i}(g+1) = \begin{cases} p_{best,i}(g), & \text{if } f(x_{i}(g+1)) \geq f(p_{best,i}(g)) \\ x_{i}(g+1), & \text{otherwise} \end{cases}
\]

(3)

and the global best position \( g_{best} \) found from all particles during the previous three steps is defined as

\[
g_{best}(g + 1) = \arg \min_{p_{best,i}} f(p_{best,i}(g+1)), \quad 1 \leq i \leq s
\]

(4)
The value of moving vector $v_i$ can be restricted to the range $[-v_{\text{max}}, v_{\text{max}}]$ to prevent particles from moving out of the search range.

3. Balanced Guide Assignment for MOPSO

Although there are numerous approaches of MOPSO, premature convergence, diversity and solutions located on/closer to the Pareto front when solving MO problems are still the main deficiency. In the original PSO (single-objective), each particle’s current moving vector refers to its $p_{\text{best}}$ and $g_{\text{best}}$ simultaneously. In single-objective problems there is only one existent $g_{\text{best}}$; but in MO problems, more than one conflicting objectives will both be optimized. The number of non-dominated solutions which are located on/near the Pareto front will be more than one. Therefore, each non-dominated solution can be the $g_{\text{best}}$ and provides its position information to current particle.

In this paper, the balanced guide assignment is introduced to MOPSO called BGA-MOPSO for picking up the various $g_{\text{best}}$ (local guide) for each particle. In order to keep diversity of non-dominated solutions, a cluster based archive method is also introduced to reserve representative non-dominated solutions from the archive.

3.1. Balanced Guide Assignment

The balanced guide assignment (BGA) will assign more particles toward to non-dominated solution with longer solution distance $d$, and assign fewer particles toward to non-dominated with shorter distance. For example, there are six particles in the population and three non-dominated solution in the archive shown in Fig. 1. Each particle has its own sorted number and so as the archive members. For BGA, each archive member will lead different numbers of particles’ as their guide.

In other word, the particle 1, 2 and 3 will be lead by archive member $N_1$; the particle 4 and 5 will be lead by archive member $N_2$, the particle 6 will be lead by archive member $N_3$. Thus, more particles will be able to discover the area around the archive member 1 to find more potential solutions. It will make solutions found with lower diversity and can avoid solution fall into local optimum.
3.2. Perturbation Method

In PSO’s searching behavior, the gbest is an important clue in leading particles to the global optimal solution. It is unavoidable for the solution to fall into the local optimum while particles try to find better solutions. In fact, after several generations, particles will gather in several clusters, or even just one cluster. Each particle in the cluster may perform a local search to follow evolution, but not be able to explore other better solutions.

In order to increase particles’ solution searching ability and explore unsearched solution space for finding more potential solutions. Hence, the perturbation method, a mutation-like evolutionary strategy, is also involved to proposed MO algorithm. A solution searching example for MO algorithm on solving minimum-minimum problems with or without perturbation operation is represented in Fig. 2.

![Fig. 2 An Solution Searching Example of MO Algorithm Involves With or Without Perturbation Method](image)

3.3. Archive

The function of the repository controller is to make decision for adding certain solutions into the archive or not. The decision-making process is stated as follows.

1. If the archive is empty, any new solution \( N_S \) found will always be accepted and stored in archive (Case 1, in Fig. 3).

2. If the new solution is dominated by any individual in the archive, then such a solution will be
discarded (Case 2, in Fig. 3).

3. If none of the solutions contained in the archive dominates the new solution, then such a solution will be stored in the archive (Case 3, in Fig. 3).

4. Otherwise, if there are solutions in the archive that are dominated by the new solution, then such dominated solutions will be removed from the archive (Case 4, in Fig. 3).

Finally, after updating all non-dominated solutions, the cluster procedure will then be activated to eliminate similar solutions for keeping lower diversity of non-dominated solutions.

3.4. Cluster Approach

After all archive members are updated by particles in current generation, some of these non-dominated solutions are with similar solution characteristic. Keeping all of them in a limited archive will affect following non-dominated solutions subsumed, due to the members in archive occupy too much space. It will also cause a poor-distributed Pareto front. To overcome this problem, the cluster approach is introduced to sift non-dominated solution found so far from the archive. First, each particle is assigned a radius of cluster $r$. Then a selected particle will become the cluster center in sequence, other solutions (similar to the cluster center) locate within a radius of $r$ will be eliminated.

![Fig. 4 (a) Archive without Cluster Operation (b) Archive with Cluster Operation](image)

The particles locate in circle of other particles’ will be remove. It should be clear that the survivors in the archive are the representative solutions set. This procedure can keep desired population diversity form the non-dominated solution set found so far. The radius of cluster $r$ is defined as follows:

$$ r = \frac{d\left[\left(f_i(x)_{\text{max}} \cdot f_j(x)_{\text{min}}\right)\left(f_i(x)_{\text{max}} \cdot f_j(x)_{\text{min}}\right)\right]}{\text{external\_repository\_size}*c} $$

where $c$ is the cluster parameter, $d$ is the Euclidean distance (measured in the objective domain), and $f_i(x)_{\text{max}}$ and $f_i(x)_{\text{min}}$ are the maximum and minimum of the $i$th objective in non-dominated solution found so far, respectively. An example of archive with/without cluster approach is shown in Fig. 4 (a) and (b). The cluster approach can preserve most significant solutions from archive. It can reserve more space to accommodate other significant solutions found latter to be stored, and also can keep diversity of solutions in archive.
4. Experiments

This section presents a six test functions and three performance metrics for MO optimization, which are adopted to compare the performance of proposed method with other popular MOEAs, include SPEA2 [7], NSGAII [9] and Sigma method [16]. The Sigma method is a PSO-based MO algorithm, and NSGA-II and SPEA2 are GA-based. In the experiments, the population size of each method is set as 100, and the maximum fitness evaluations (FEs) are set as 300,000.

4.1. Test Functions

The chassed six test functions, including ZDT1, ZDT2, ZDT3, ZDT4[23], SCH[24] and FON [25] which are designed by Zitzler et al. These problems include characteristics that are suitable for examining the effectiveness of MO approaches in maintaining the population diversity as well as converging to the Pareto front. The detail of these test functions are listed in Table 1.

<table>
<thead>
<tr>
<th>Test Functions</th>
<th>Dimensions (N)</th>
<th>Boundary</th>
<th>Objective Functions</th>
</tr>
</thead>
<tbody>
<tr>
<td>ZDT1</td>
<td>30</td>
<td>[0, 1]</td>
<td>( f_i(x) = x_i )</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>( f_2(x) = g(x)\left[1 - \sqrt{x_i / g(x)}\right] )</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>( g(x) = 1 + 9\sum_{i=1}^{N} x_i / (N-1) )</td>
</tr>
<tr>
<td>ZDT2</td>
<td>30</td>
<td>[0, 1]</td>
<td>( f_i(x) = x_i )</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>( f_2(x) = g(x)\left[1 - (x_i / g(x))^2\right] )</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>( g(x) = 1 + 9\sum_{i=1}^{N} x_i / (N-1) )</td>
</tr>
<tr>
<td>ZDT3</td>
<td>30</td>
<td>[0, 1]</td>
<td>( f_i(x) = x_i )</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>( f_2(x) = g(x)\left[1 - \sqrt{x_i / g(x)} - \frac{x_i}{g(x)}\sin(10\pi x_i)\right] )</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>( g(x) = 1 + 9\sum_{i=1}^{N} x_i / (N-1) )</td>
</tr>
<tr>
<td>ZDT4</td>
<td>10</td>
<td>[0, 1]</td>
<td>( f_i(x) = x_i )</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>( f_2(x) = g(x)\left[1 - \sqrt{x_i / g(x)}\right] )</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>( g(x) = 1 + 10(N-1) + \sum_{i=1}^{N} [x_i^2 - 10\cos(4\pi x_i)] )</td>
</tr>
<tr>
<td>SCH</td>
<td>1</td>
<td>[-10³, 10³]</td>
<td>( f_i(x) = x_i^2 )</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>( f_2(x) = (x - 2)^2 )</td>
</tr>
<tr>
<td>FON</td>
<td>3</td>
<td>[-4, 4]</td>
<td>( f_i(x) = 1 - \exp\left(-\sum_{j=1}^{n} (x_j - \frac{1}{\sqrt{N}})^2\right) )</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>( f_2(x) = 1 - \exp\left(-\sum_{j=1}^{n} (x_j + \frac{1}{\sqrt{N}})^2\right) )</td>
</tr>
</tbody>
</table>

4.2. Performance Metrics

The three different quantitative performance metrics for MO optimization are employed. These metrics are capable of evaluating non-dominated individuals in several nontrivial aspects and have been widely used in the studies of MOEAs.
1) **Generational Distance (GD):** The concept of generational distance was introduced by Van Veldhuizen and Lamont [26]. It is a way to estimate the Euclidean distance between the member in non-dominated solution found so far and its nearest member in Pareto optimal set and is defined as

\[
GD = \frac{1}{s} \sqrt{\sum_{i=1}^{s} d_i^2}
\]  

where \(s\) is the number of member in the set of non-dominated solution found so far and \(d_i\) is the Euclidean distance (measured in the objective domain). GD = 0 indicates all the elements found are in the Pareto optimal set.

2) **Diversity (\(\Delta\)):** The metric of diversity was proposed by Deb et al. It measures the extent of spread achieved among the obtained solutions and is defined as

\[
\Delta = \frac{d_f + d_i + \sum_{i=1}^{s} |d_i - \bar{d}|}{d_f + d_i + (s-1)d}
\]

where the parameter \(d_f\) and \(d_i\) are the Euclidean distances between the extreme solutions and the boundary solutions of the obtained non-dominated set. The parameter \(\bar{d}\) is the average of all distances \(d_i, i = 1, 2, \ldots, (s-1)\), assuming that there are \(s\) solutions on the best non-dominated front [27].

3) **Maximum Spread (MS):** The metric of maximum spread [28] measure the

\[
MS = \sqrt{\frac{1}{n} \sum_{i=1}^{n} \left( \frac{\min(f^{\max}, F^{\max}) - \max(f^{\min}, F^{\min})}{F^{\max} - F^{\min}} \right)^2}
\]

where \(n\) is the number of objective, \(f^{\max}\) and \(f^{\min}\) are the maximum and minimum of the \(i\)th objective in non-dominated solution found so far , respectively; and the \(F^{\max}\) and \(F^{\min}\) are the maximum and minimum of the \(i\)th objective in Pareto optimal set, respectively.

Summarizing the above statements, a nice MO optimizer should perform shorter generational distance (GD), lower diversity (\(\Delta\)), and also results maximum spread (MS) equal or closer to 1 but not greater than.

### 4.3. Experiment Results

The Perturbation rate of the BGA-MOPSO was set as 1. Fig. 5 presents the box plot of 30 runs of four algorithms on the six test functions. For the GD metric, the BGA-MOPSO achieved better result than other three algorithms on ZDT1, ZDT2, ZDT3 and ZDT4. Sigma method performs better on SCH and FON, and the proposed method results very closer to sigma method on SCH. Most solution found by the BGA-MOPSO was very closer to the Pareto front. For the results of the diversity metric, the BGA-MOPSO performed better in all test functions. Lower diversity represents that the solution set has better solution distribution. Finally, for the results of the maximum spread metric, the BGA-MOPSO achieved better result than other three algorithms on the MS metric on ZDT1, ZDT2, ZDT3, ZDT4 and SCH. The sigma method performs better on FON.
5. Conclusions

This paper presents a density based local guide assign approach called balanced guide assignment for multi-objective particle swarm optimization (BGA-MOPSO). The strategy can significantly assign a suitable pbest to each particle for leading them toward to potential searching space. It can improve particles’ searching abilities and easier for finding better solutions which locate on/near to the Pareto front. The Perturbation method will make PSO more robust and will prevent particles from falling into the local minimum. Furthermore, the cluster based archive can efficiently utilize storage space of archive, reserve significant solutions only, and also keep diversity of solutions stored in it.

Six test functions were adopted for experiments. It shows that the BGA-MOPSO can find better solutions with shorter GD, higher spread and lower diversity than SPEA2, NSGAII and Sigma method on the problems studied.

Fig. 5 Box Plots for the Metrics of Generational Distance (GD), Diversity ($\Delta$) and Maximum Spread (MS)
References


